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Short Communication

Estimation of the Dispersion of Compressional Waves in Rocks From Ultrasonic to Seismic Frequencies*

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In the investigation of the composition of the crust and the mantle of the earth, velocities measured in the laboratory are compared to seismic wave velocities. Because of dispersion due to anelasticity, question has been raised as to the validity of such comparision (Liu et al., 1976). The laboratory velocities are indeed obtained at ultrasonic frequencies of a few megacycles per second while the seismic velocities at frequencies of a few cycles per second. To date no laboratory velocity measurements on the same sample and at these two extreme frequencies have ever been made to determine the amount of dispersion. In this note, we have made an estimation of the dispersion of compressional waves in a very fine grain limestone. The amount of dispersion was calculated from the internal friction data.

It has been shown that, in a linearly attenuating body, the velocity at a given frequency is uniquely determined from the knowledge of the internal friction $Q^{-1}$ of the body over all frequencies, and vice versa (see e.g. Futterman, 1962). The velocity dispersion relation can be written (Kanamori and Anderson, 1977):

$$\frac{1}{v(f_i)} = \frac{1}{v(\infty)} \left(1 + \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{Q^{-1}(f_i)}{f-f_i} df \right)$$

(1)

where $f$ is frequency, $v(f_i)$ the velocity at frequency $f_i$, $v(\infty)$ the velocity at infinite frequency and $P$ stands for Cauchy principal value of the integral. The applicability of this relation is limited by the lack of knowledge of $Q^{-1}$ over all frequencies. $Q^{-1}$ has been measured on rocks at frequencies of a few hertz to a few megahertz, but generally over a narrow frequency band for a particular sample.

For our calculation, we have taken from the literature a rock for which detailed $Q^{-1}$ data are available over the widest frequency band. This is a very

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fine grain Solenhofen limestone, whose internal friction for longitudinal vibrations was measured at ambient conditions from 30 kHz to 5 MHz (Mason, 1971). Given the small grain size of the rock (average grain size 9 μm), absorption due to scattering was considered negligible to a frequency of 5 MHz (Mason, 1971). According to Mason (1971), we have fitted the following equation to the experimental \( Q^{-1} \) data (Fig. 1):

\[
Q^{-1}(f) = \frac{2.54 \cdot 10^{-3} + 5.57 \cdot 10^{-8} f}{1 + 1.31 \cdot 10^{-12} f^2}
\]  

(2)

Equation (2) is used for a rough estimation of \( Q^{-1} \) outside the frequency band of the actual measurements. At low frequencies (1 Hz to 10 kHz), the estimated \( Q^{-1} \) (2.5 \( \cdot \) 10\(^{-3}\)) falls within the values of \( Q^{-1} \) measured on various limestones, as given by Bradley and Fort (1966). These values range from 1 \( \cdot \) 10\(^{-3}\) to 5 \( \cdot \) 10\(^{-3}\). The high values of \( Q^{-1} \) correspond to rocks with high microcrack density. No \( Q^{-1} \) data are available below 1 Hz for limestone. Published data on oxides (see Jackson and Anderson, 1970) indicate that, at very low frequencies (below 10\(^{-5}\) Hz), \( Q^{-1} \) may be large. The effect of large absorption at low frequencies will be discussed below. At high frequencies, absorption due to scattering is dominant but is not considered here, as it does not introduce dispersion.

In order to satisfy the requirement that \( Q^{-1} \) be an odd function of frequency (see Futterman, 1962), equation (2) is truncated at \( f = 0 \), and for \( f < 0 \), \( Q^{-1} \) is taken to be equal to:

\[
Q^{-1}(f) = -\frac{2.54 \cdot 10^{-3} - 5.57 \cdot 10^{-8} f}{1 + 1.31 \cdot 10^{-12} f^2}
\]  

(3)
Using Eqs. (1), (2), and (3), the percentage difference between the velocities at the frequencies \( f_1 \) and \( f_2 \) is given by:

\[
d = \frac{B_2 - B_1}{1 + B_1} \cdot 10^2
\]  
(4)

where \( B_1 \) and \( B_2 \) are the values of

\[
\frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{Q^{-1}(f)}{f - f_i} df
\]

at \( f_i \) equal to \( f_1 \) and \( f_2 \) respectively. For \( f_1 = 1 \text{ MHz} \) and \( f_2 = 1 \text{ Hz} \) for instance, \( d \) is found to be 2.5.

If at frequencies below 1 Hz, the actual \( Q^{-1} \) is larger than the estimated one, as mentioned above, the value of \( d \) should be smaller than 2.5. Moreover, at high pressure, \( Q^{-1} \) decreases due to grain-boundary microcrack closure (Birch and Bancroft, 1938; Walsh, 1966). Birch and Bancroft data (1938) for torsional vibrations in Solenhofen limestone show a decrease in \( Q^{-1} \) by a factor of about 2 from 0.2 to 4 kbar, at a frequency of the order of a few kHz. Such a decrease in \( Q^{-1} \) would further reduce the value of \( d \). Thus the 1 MHz velocity would differ from the 1 Hz velocity by at most 2.5%. It is noteworthy that, in the present limestone, the frequency interval, 1 Hz–1 MHz, is outside the frequency domain of the unrelaxed elastic state.

Knowledge of \( Q^{-1} \) for other rock types is not as detailed as that of the present limestone. However, values of \( Q^{-1} \) measured at ambient conditions on a few mafic and ultramatic rocks, at frequencies between 1 Hz and \( 10^5 \) Hz, also fall in the \( 10^{-3} \) range (Bradley and Fort, 1966). Measurements by Birch and Bancroft (1938) on nine different rock types at a frequency of a few kHz, at room temperature and under a pressure of 4 kbar, show a variation in \( Q^{-1} \) by a factor of about three from one rock type to another. One of the highest value of \( Q^{-1} \) they have reported is that of a Solenhofen limestone. At megacycle frequencies, Mason (1971) found comparable \( Q^{-1} \) for longitudinal vibrations in slate and limestone. His granite shows higher \( Q^{-1} \), but this is presumably due to large microcrack density. On the basis of the above data, the internal friction and therefore the velocity dispersion for varied rock types should not differ much from those for the Solenhofen limestone considered here.

In this note, we have made an estimation of the dispersion of the compressional wave velocity in rocks from 1 Hz to 1 MHz. The dispersion was calculated from laboratory \( Q^{-1} \) data collected at ambient conditions. To date, no high temperature laboratory \( Q^{-1} \) data over a wide frequency band is available. The effect of temperature, however, is to decrease the viscoelastic relaxation time. This results in a shift of the high frequency limit of the relaxed elastic domain towards higher frequencies and a decrease in the elastic wave velocity dispersion in the frequency band of interest here. On the basis of the laboratory \( Q^{-1} \) data for the Solenhofen limestone studied here, the velocities at 1 Hz and 1 MHz is estimated to differ by less than 2.5%. Thus velocities measured at 1 MHz in laboratories are directly comparable to seismic wave velocities measured at a frequency of 1 Hz.
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