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Letter to the Editor

Errors in Matrix Element Expressions for the Reflectivity Method

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The purpose of this note is to correct some long standing errors in expressions for the bottom half space delta matrix used in the computation of the propagator matrix of the transition zone in the reflectivity method. The expressions were first given by Fuchs (1968) in his Sect. 6.3, and the same erroneous expressions have reappeared in Kind (1976).

Using the notation of Fuchs (1968), the lower half space matrix for an $n-2$ layer transition zone between 2 half spaces is:

$$T_n^{-1}=F\cdot\begin{pmatrix}-j2\mu_n k v_n v_n' & +j\mu_n l_n v_n' & -v_n v_n' & -kv_n' \\ -j\mu_n l_n v_n & -j2\mu_n k v_n v_n' & -kv_n & +v_n v_n' \\ -j2\mu_n k v_n v_n' & -j\mu_n l_n v_n' & -v_n v_n' & +kv_n' \\ +j\mu_n l_n v_n & -j2\mu_n k v_n v_n' & +kv_n & +v_n v_n'\end{pmatrix} \quad (1)$$

where

$$F=\frac{\beta_n^2}{2\mu_n v_n v_n' \omega^2}. \quad (2)$$

The elements of the delta matrix of the lower half space are the 2×2 subdeterminants of the T_n^{-1} matrix. The correct expressions are:

$$\hat{t}^n|_{12}^{12}=\hat{t}|_1^1=-\frac{\beta_n^4}{4\omega^4}\cdot\left(4k^2+\frac{l_n^2}{v_n v_n'}\right)$$

$$\hat{t}^n|_{13}^{12}=\hat{t}^n|_2^1=\frac{j\beta_n^2}{4\mu_n v_n'\omega^2}$$

$$\hat{t}^n|_{14}^{12}=\hat{t}^n|_3^1=\hat{t}^n|_{23}^{12}=\hat{t}^n|_4^1=\frac{-j\beta_n^4}{4\mu_n\omega^3 c}\cdot\left(2+\frac{l_n}{v_n v_n'}\right) \quad (3)$$

$$\hat{t}^n|_{24}^{12}=\hat{t}^n|_5^1=-\frac{j\beta_n^2}{4\mu_n v_n\omega^2}$$

$$\hat{t}^n|_{34}^{12}=\hat{t}^n|_6^1=-\frac{1}{4\rho_n^2\omega^4}\cdot\left(1+\frac{k^2}{v_n v_n'}\right).$$

These are the only elements of the 6×6 delta matrix for the lower half space that are required for computing reflection and transmission coefficients of the transition zone. Equivalent expressions for the half space delta matrix elements are also given by Watson (1970). They differ from the expression in (3) because Watson (1970) uses the dilatational and rotational wave solutions given by Haskell (1953) rather than the potential solutions given by Fuchs (1968).

The expressions given by Fuchs (1968) and Kind (1976) were apparently obtained by evaluating the 2×2 subdeterminants of the matrix in the brackets in (1) and multiplying by the common factor F in (2). In fact, the subdeterminants of the matrix in brackets should have been multiplied by F^2. Therefore, all delta matrices $m|_{kl}^{ij}$ of the Haskell propagator matrix M computed with the expressions in Fuchs (1968) and Kind (1976) would have a multiplicative error of F.

It should also be pointed out that the definitions of the vertical wavenumbers v_i and v_i' given by Kind (1976) are incorrect. The correct definitions are given in (12) and (13) in Fuchs (1968).

Fortunately, none of the results of the application of the reflectivity method to date are invalidated (Fuchs and Müller 1971; Kennett 1972; Kind and Müller 1975, to name a few). This is because these applications have required only the computation of topside reflection coefficients from the transition zone. For example, the reflection coefficients for incident P waves on the top of the transition zone are (Fuchs 1968; Červený 1974):

$$R_{PP}=R_{11}=m|_{23}^{12}/m|_{12}^{12}$$
$$R_{PS}=R_{12}=-m|_{13}^{12}/m|_{12}^{12}. \quad (4)$$

Since the numerators and denominators are in error by a factor of F, the errors are removed by division. The error does not divide out for other coefficients, however. For the case of an incident P wave from the top, the top to bottom transmission coefficients are (Fuchs 1968; Červený 1974):

$$T_{PP}=R_{13}=m_{31}\cdot R_{PP}+m_{32}\cdot R_{PS}+m_{33}$$
$$T_{PS}=R_{14}=m_{41}\cdot R_{PP}+m_{42}\cdot R_{PS}+m_{43}$$

The m_{ij} are elements of the Haskell matrix M. For these expressions, the errors remain in the elements of the M matrix and are not divided out. From Červený (1974), the expressions for R_{13}, R_{14}, R_{23}, R_{24}, R_{31}, R_{32}, R_{41}, and R_{42} would also contain the multiplicative error whereas in the expressions for R_{21}, R_{22}, R_{33}, R_{34}, R_{43}, and R_{44}, the error is removed by division.

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Comment of the Editor

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