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Two Methods of Solving the Linearized Two-Dimensional Inverse Seismic Kinematic Problem

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Abstract. In a laterally variable seismic medium, the slowness function $s(x, z)$ can be derived from travel-time curves of surface-to-surface refracted waves if the arrival times are interpreted in the form of a two-dimensional function. In the following article two methods are suggested for solving the inverse kinematic problem using a linearized formulation. The arrival times $t(p, q)$ are arranged according to increasing epicentral distance q, and p represents the x-coordinate of the point midway between source and receiver. The first method is based on the Fourier transform of the time function $t(p, q)$ in the variable p. This method can be applied to a system of travel-time curves with epicentral distances small in comparison to the length of the profile interval x_p, i.e., $q \ll x_p$. The second, grid method interprets recursively the arrival times $t(p_j, q_i)$ from the smallest epicentral distance to the largest one. The results of one interpretation step are the grid values of the slowness function $s(x, z)$ on one grid line $z = z_i$. In contrast to the method of Fourier coefficients the recursive grid method does not introduce any limitation of the shape of the solution region. Both methods have been tested using both theoretical and real data.

Key words: Inverse kinematic seismic problem – Laterally inhomogeneous media – Ray interpretation of travel-times

Introduction

The interpretation of travel-time curves of surface-to-surface refracted waves represents one possible method for estimating the velocity distribution in deep-seated structures. If the medium is assumed to be inhomogeneous in the vertical direction only, then the unknown velocity function $v(z)$ of depth can be obtained by the well-known Wiechert-Herglotz integral transformation of the travel-time curve of the surface-to-surface refracted wave $t(r)$. In the majority of computational algorithms based upon the Wiechert-Herglotz theory it is assumed that $t(r)$ is a one-valued function of the epicentral distance r with non-increasing derivative, even though, as early as 1932, the theory was generalized by Slichter (1932) to the case of loop-like travel-time curves occurring in a medium with strongly varying velocity gradient or even with velocity discontinuities. In this case the travel-time curve of the supercritically reflected wave must also be taken into account. The problem of the estimation of the function $v(z)$ in the presence of a low-velocity channel was studied in a paper by Gerver et al. (1966). Practical difficulties associated with the travel-time curve ambiguity meant that a new theory was needed and this has recently appeared in the form of the tau method (Bessonova et al. 1974), sometimes called the method of extremal inversion (Kennett 1976). The tau method is based on the Legendre transform of the travel-time curve $t(r)$ that transforms the loop-like travel-time curve into the one-valued function $\tau(p)$ (Gerver et al. 1967). This approach also allows the setting of reasonable bounds to variations of the velocity function $v(z)$, deduced from the character of the discretely measured values $t(r)$. In particular, incompletely recorded travel-time curves, as well as the discontinuous ones that are met frequently in interpretation of deep seismic sounding data, can be more reliably interpreted by means of the tau method (Beránek et al. 1979).

Real seismic media are generally also inhomogeneous in the horizontal directions x and y. If the inhomogeneity is negligible in one direction (for instance y), it then makes sense to consider the two-dimensional inverse problem of finding the velocity distribution $v(x, z)$. It is now assumed that an adequately dense system of travel-time curves $t(x_S, x_R)$ is available with sources $S = (x_S, 0)$ and receivers $R = (x_R, 0)$ distributed uniformly over the entire interval under consideration $\langle 0, x_p \rangle$ along the profile (Fig. 1).

Instead of the variables x_S, x_R it is more advantageous to operate with other variables p, q defined by the relations

$$p = (x_S + x_R)/2, \quad q = |x_R - x_S|. \tag{1}$$

This new coordinate frame is also currently used in the CDP method of seismic reflection prospecting. The p variable represents the x coordinate of the midpoint between the source S and the receiver R. The variable q stands for the epicentral distance (offset of measurement). The function $t(p, q)$ then represents, for the constant offset variable q, an isoline of the so called special time field (Puzyrev et al. 1975). The use of the variables p, q is based on the equivalence of the travel-times of seismic waves between reciprocal points, i.e., $t(x_R, x_S) = t(x_S, x_R)$. Since the reciprocal measurements are often performed in practice, we remove this redundancy in the input data.

While methods of solution of the one-dimensional inverse problems are relatively well developed and can thus be immediately applied to solve practical problems (Bessonova et al. 1976; Kennett 1976), the study of multi-dimensional problems is still at the beginning. The inverse kinematic problem for more dimensions is, from the mathematical point of view, a classical ill-posed problem. Its solution has required a new

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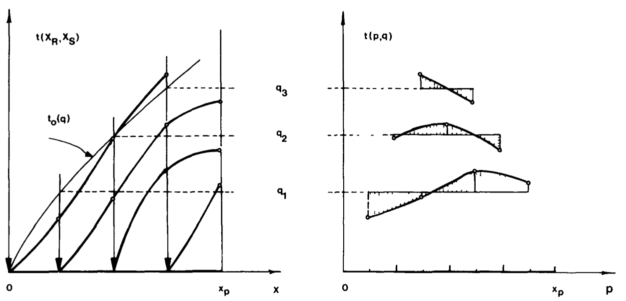

Fig. 1. Two representations of arrival times of surface-to-surface refracted waves. System of travel-time curves $t(x_R, x_S)$ with a constant coordinate x_S of the source S and a variable coordinate x_R of the receiver point R can be arranged into time isolines $t(p, q)$ with a constant offset of measurements $q = |x_S - x_R|$ and with the lateral variable $p = (x_R + x_S)/2$. Travel-time curve $t_0(q)$ is obtained as the average of all arrival times due to common offset q

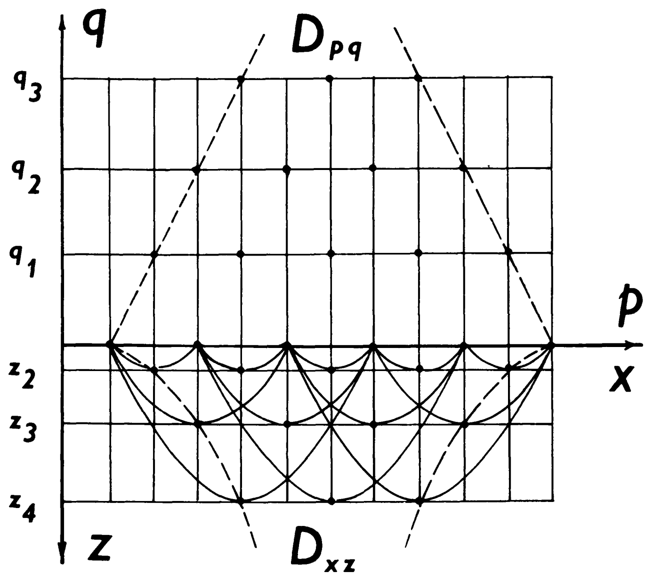

Fig. 2. All grid points (x_j, z_i) in the solution region D_{xz} are defined by the vertices of supporting ray paths $\Gamma_0(p_j, q_i)$, i.e., $x_j = p_j$, $z_i = \xi(q_i)$. The form of the region D_{xz} implies the useful part D_{pq} of input data in the (p, q) coordinate plane

approach to the question of solution stability (Romanov 1972). The first strict mathematical formulation of the two-dimensional inverse problem as a certain Cauchy problem for the non-linear differential equation of the Hamilton-Jacobi type has been presented by Belonosova and Alexeyev (1967). The travel-time curve of the surface-to-surface refracted wave $t(x_S, x_R)$ and its partial derivatives $\partial t/\partial x_S$, $\partial t/\partial x_R$ represent initial conditions. It has been shown by theoretical examples (Belonosova and Alexeyev 1967) that the finite-difference approximation yields a numerical solution to the problem with an accuracy of 5-10%.

Another approach (Romanov et al. 1978) is based on a combination of solutions of both the inverse and the direct problem. Both approaches require both the two dimensional travel-time curve $t(x_S, x_R)$ and its partial derivatives. However, commonly used systems of measurements do not yield a sufficiently dense set of data that would make possible a fair approximation of the required initial data. In particular, derivatives $\partial t/\partial x_S$ and $\partial t/\partial x_R$ derived from sparsely measured travel-times $t(x_S, x_R)$ would cause considerable inaccuracies in the results obtained by these methods. In such a case, the simplified linearized formulation of the inverse kinematic problem appears as one way out.

The linearization of the inverse problem in a multi-dimensional case has been suggested by Romanov (1972). It represents a particular generalized analogue of the linearized approach to solving one-dimensional problems (Johnson and Gilbert 1972).

Let us consider a two-dimensional problem. The following expression is valid for the travel-time of seismic energy propagated from point S to point R along trajectory $\Gamma(R, S)$ (Fig. 2):

$$t(R, S) = \int_{\Gamma(R,S)} s(x, z)\, d\gamma \tag{2}$$

where the slowness function $s(x, z)$ represents the reciprocal of the velocity function $v(x, z)$ in the medium. In agreement with the Fermat principle, the curve $\Gamma(R, S)$ connecting points R and S represents the geodetic in the Riemann space with the metric $s(x, z)$. To solve the inverse kinematic problem means, in the geometrical interpretation, to find the unknown metric $s(x, z)$ by means of the metrical distances $t(R, S)$ known between all boundary points S and R of the region D_{xz} under consideration. It is obvious that the expression (2) cannot be inverted with respect to the unknown metric $s(x, z)$ because this metric also defines the geodetics along which the integration is performed. The process of linearization suggested by Romanov (1972) consists in dividing the metric $s(x, z)$ into a regular part $s_0(x, z)$ and a perturbative part $\Delta s(x, z)$, i.e., $s(x, z) = s_0(x, z) + \Delta s(x, z)$, provided that the norms of the functions $s(x, z)$ and $s_0(x, z)$ in the region D_{xz} satisfy the condition $\|\Delta s(x, z)\| \ll \|s_0(x, z)\|$. It can then be proved on the basis of (2) that the travel-time also consists of the regular part $t^0(R, S)$ and the perturbative part $\Delta t(R, S)$, i.e.,

$$t(R, S) = t_0(R, S) + \Delta t(R, S), \tag{3}$$

where $t_0(R, S)$ corresponds to the travel-time of wave propagation along the geodetic $\Gamma_0(R, S)$ generated by the metric $s_0(x, z)$. For the perturbative part it follows that

$$\Delta t(R, S) = \int_{\Gamma_0(R,S)} \Delta s(x, z)\, d\gamma_0 + \Delta t_2(R, S). \tag{4}$$

The term $\Delta t_2(R, S)$ approaches zero as a quantity of 2nd order with respect to $\Delta t(R, S)$ if the norm $\Delta s(x, z)$ goes to zero. If the term Δt_2 is neglected in expression (4), the relation between Δt and Δs is linear because the geodetic $\Gamma_0(R, S)$ appearing in expression (4) does not depend on Δs. Expression (4), or its differential analogue (Novotný 1980), serves as a base for all algorithms of numerical solution of the linearized inverse kinematic problem.

The Method of Fourier Coefficients

Relation (4) can be used either for the removing of arrival-time deficiency relative to some velocity model obtained in another way (Romanov et al. 1978) or in an iterative mode to find the unknown velocity model with desired accuracy. In the case of weak lateral inhomogeneity we can restrict ourselves to the first iteration and choose the starting model independent of the lateral variable. Romanov (1972) has suggested an algorithm on the basis of expression (4) for a spherically symmetrical starting model, i.e., $s_0(\rho, \varphi) = s_0(\rho)$; ρ,

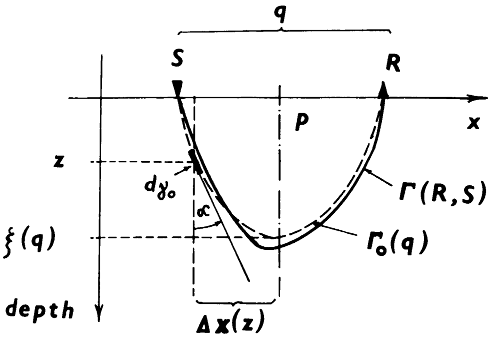

Fig. 3. In a laterally homogeneous medium neither the ray path $\Gamma_0(q)$ nor its characteristics $s_0(z)$, $\Delta x(z)$ and ξ depend on the lateral coordinate p

φ are polar coordinates. The unknown function $\Delta s(\rho, \varphi)$ was sought in the form of finite polynomial expansions in $\exp(i\varphi)$ and ρ. After integration according to Eq. (4), an expression was obtained for $\Delta t(R,S)$ which is linearly dependent on the unknown expansion coefficients. If a satisfactory number of arrival times $t(R,S)$ is available, optimal values of coefficients can be found by the least-squares procedure. Earthquake records along the Pamir-Baykal profile have been interpreted in this way (Alexeyev et al. 1971).

Besides this algorithm, another method of solving the inverse problem is suggested by Romanov (1972). The integral equation relating the Fourier spectra of Δt and Δs functions with respect to the lateral variable p was utilized (Eq. (1)). On the basis of integral equation properties the uniqueness of solution of the inverse linearized problem was proved. However, no numerical algorithm based on the integral equation was suggested.

The Fourier coefficient method presented in this section is based on an expansion of Δt and Δs functions in the lateral variable p by means of Fourier series. This allows derivation of directly solvable system of linear equations for the unknown coefficients of the Fourier series for Δs instead of the integral equation mentioned above.

Let us assume a starting model independent of the lateral variable x. Thus, in a Cartesian coordinate frame we have $s_0(x,z) = s_0(z)$. It is then possible to transform the integration along the geodetics $\Gamma_0(R,S)$ in expression (4) into a simple integral with respect to the variable z:

$$\Delta t(p,q) = \int_0^{\xi(q)} [\Delta s(p - \Delta x(z), z) + \Delta s(p + \Delta x(z), z)] \frac{dz}{\cos\alpha(z)}. \qquad (5)$$

Here we used new variables p and q defined by Eq. (1). The maximum depth of the ray path $\Gamma_0(R,S)$ is denoted by ξ, $\alpha(z)$ is the angle between the ray path and the vertical at an arbitrary point (x,z). The quantity $\Delta x(z)$ (Fig. 3) is given by the following relation:

$$\Delta x(z) = \int_0^z dz'/\tan\alpha(z').$$

Time differences $\Delta t(p,q)$ are determined for individual offsets q_i as the difference between the total values of time isolines $t(p,q_i)$ and the arrival time $t_0(q_i)$ corresponding to the laterally homogeneous starting model $s_0(z)$. It thus follows $(i = 1, 2, \ldots, M)$

$$\Delta t(p,q_i) = t(p,q_i) - t_0(q_i). \qquad (6)$$

The unknown perturbative part of the metric $\Delta s(x,z)$ will be now approximated in the region $0 \leqq x \leqq x_p$, $0 \leqq x \leqq z_{\max}$ by the finite expansion $(n = -(N-1)/2, \ldots, -1, 0, 1, \ldots, (N-1)/2)$

$$\Delta s(x,z) = \sum_j \sum_n A_{nj} P_j(z)\, e^{i 2\pi n x / x_p}, \qquad (7)$$

where $P_j(z)$, $j = 1, 2, \ldots, L$ is a suitable polynomial basis. Now, we want to derive equations for the unknown Fourier coefficients A_{nj}. Therefore, we expand the left-hand side of Eq. (5) in the single variable $p \in \langle 0, x_p \rangle$

$$\Delta t(p,q) = \sum_n B_n(q)\, e^{i 2\pi n p / x_p}. \qquad (8)$$

Finally, we obtain from (5), for Fourier coefficients A_{nj} and $B_n(q)$ of expansions (7) and (8), the following linear relation

$$B_n(q) = \sum_j A_{nj} F_{nj}(q), \qquad (9)$$

where

$$F_{nj}(q) = 2 \int_0^{\xi(q)} P_j(z) \cos[2\pi n \Delta x(z)/x_p]\, dz. \qquad (10)$$

Expression (9) represents N linear systems from which all Fourier coefficients A_{nj} can be determined in terms of $B_n(q)$ and the starting slowness function $s_0(z)$. Each system consists of as many equations as time isolines are available with different offsets $(q = q_i,\ i = 1, 2, \ldots, M)$. If the number N of the polynomials used in (7) is equal to the number of time isolines M, i.e., $L = M$, there is one solution of (9). If $L < M$, there is no solution satisfying all equations, but we can perform an optimization process, finding the vector A_{nj} which gives the minimum value of the expression

$$\sum_i |B_n(q_i) - \sum_j F_{nj}(q_i) A_{nj}|^2. \qquad (11)$$

Both these possibilities have been included in the numerical realization (Novotný et al. 1979) of the algorithm which enables us to choose the maximum degree L of polynomials independently of the number M of given offsets. The algorithm was tested on an example of a theoretical model with constant velocity gradients.

To characterize the accuracy that is achieved by a linearized method in the case of theoretical models, we used the quantity $\eta(z)$, defined as a ratio of the integral absolute error of the method and the integral absolute lateral deviation of model $s(x,z)$ from the starting model $s_0(z)$ at the depth considered, i.e.

$$\eta(z) = \int_{x_1(z)}^{x_2(z)} |\Delta s_C(x,z) - \Delta s_T(x,z)|\, dx \Big/ \int_{x_1(z)}^{x_2(z)} |\Delta s_T(x,z)|\, dz. \qquad (12)$$

Indices C, T denote successively computed or theoretical values in the interval of interest $\langle x_1(z), x_2(z) \rangle$. It was found that two factors limit the application of the Fourier coefficient method:

(i) Adequacy of the fitting of the input values $\Delta t(p_j, q_i)$ by means of the expansion (8). Figure 4 illustrates the accuracy achieved by the Fourier coefficient method with respect to the different ways of fitting. Curve (1) represents the relative accuracy $\eta(z)$ of the method in the case when the Fourier coefficients were calculated from experimental values $\Delta t(p_i)$ with the help of the discrete Fourier transform. As we could use only a limited number N of the Fourier coefficients ($N \leqq 128$), we achieved a better accuracy using the finite Fourier series that approximates the course of $\Delta t(p)$ in the least-squares fashion. Further, we tested the influence of surface velocities on the accuracy of the method. The curve (2a)

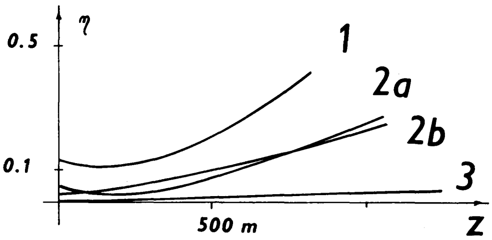

Fig. 4. Curves $\eta(z)$ characterize the relative accuracy of various linearized methods of solving the inverse problem. All the methods were tested for one particular velocity model with constant gradients. Curves **1**, **2a**, **2b** characterize the accuracy of the method of Fourier coefficients, curve **3** the method of direct inversion

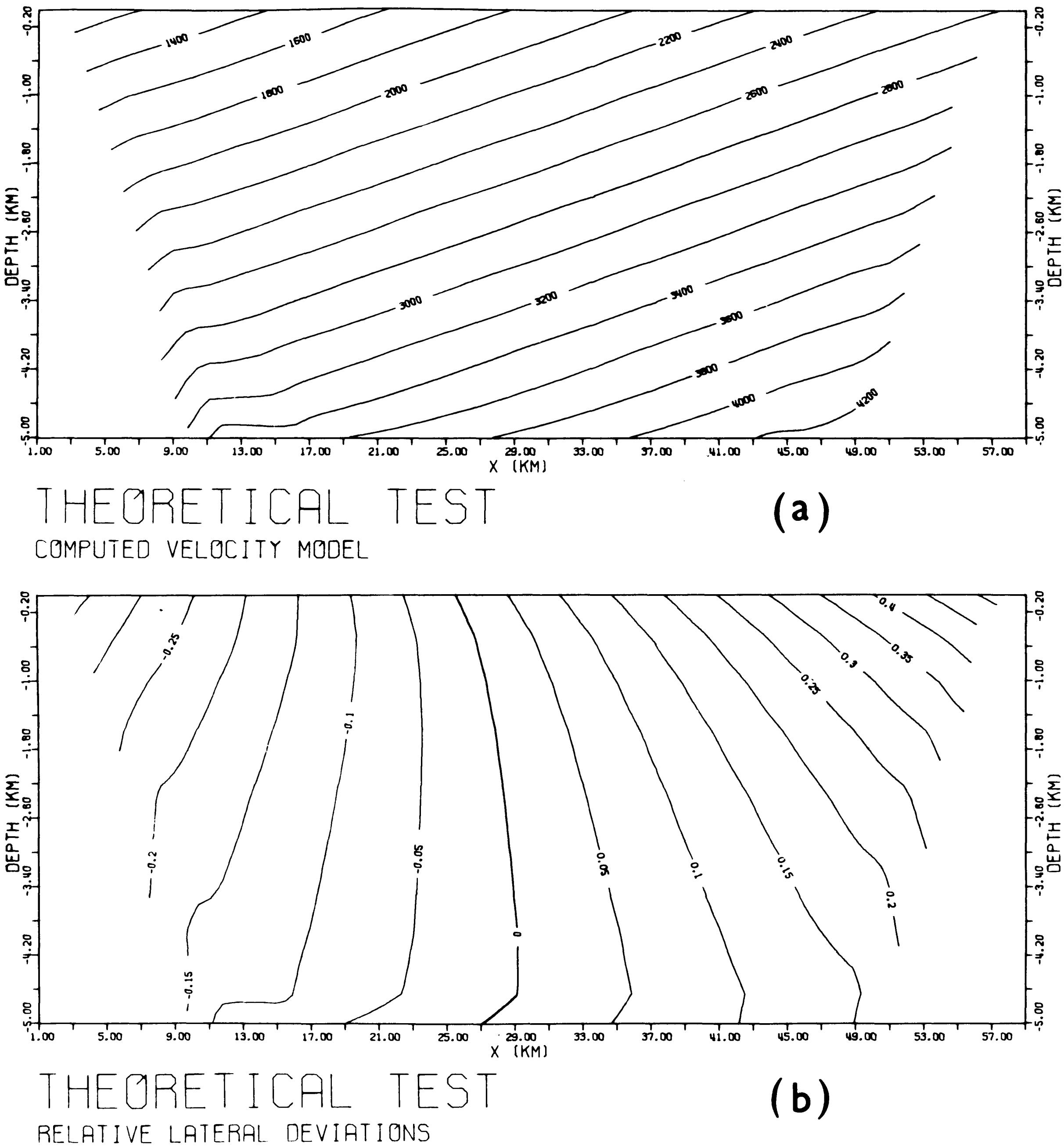

Fig. 5a–b. The test of the direct inversion method for the velocity model $v(x,z)=v_0+g(x\sin\alpha+z\cos\alpha)$, $v_0=1$ km/s, $\alpha=4°$, $g=4.10^{-4}\text{s}^{-1}$
a Contour map of velocity distribution calculated on the basis of corresponding time isolines

$$t(p,q)=\frac{1}{v_0 g}\ln\left|\frac{E_1-gq}{E_1+gq}\right|,\quad E_1=\sqrt{E_2+(qg)^2},\quad E_2=4(1+pg\sin\alpha)^2-(qg\sin\alpha)^2.$$

b Contour map of relative deviations between the starting model $s_0(z)$ and the calculated one $s(x,z)$

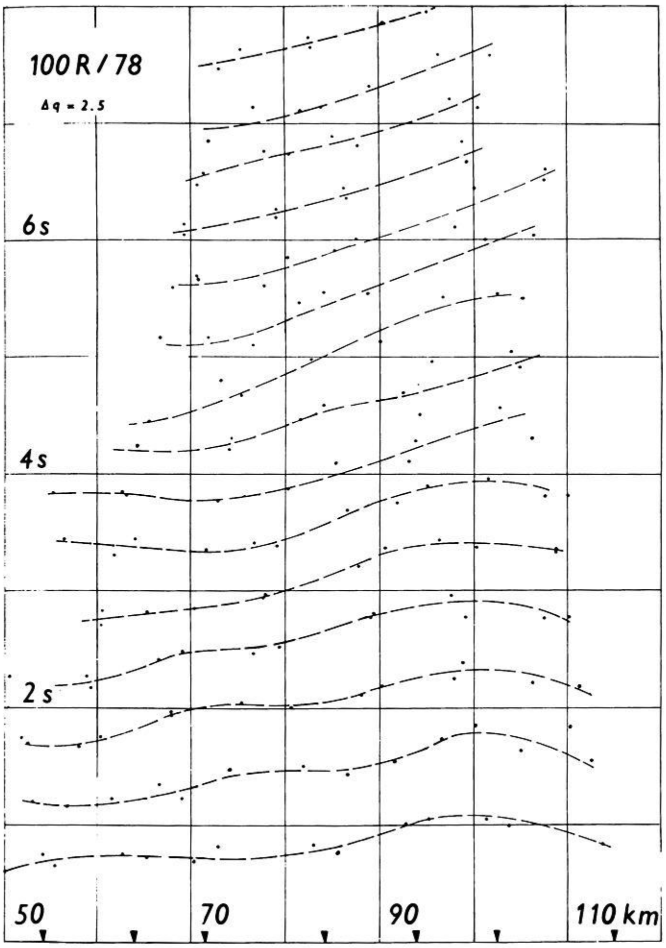

Fig. 6. Time isolines $t(p, q_i)$ obtained from the data, recorded along the Carpathian profile 100 R. For the calculation of the velocity section isolines are given with offsets $q_i = i\Delta q$, $i = 1, 2, 4, 6, 8, 11, 13$, and 15, $\Delta q = 2.5$ km

corresponds to the case when the surface velocities are taken into account, while the curve (2b) stands for the relative accuracy $\eta(z)$ in the case when the method works without the input of the surface velocities. We observe that the method is rather insensitive to this influence.

(ii) The shape of the region D_{pq} where values $\Delta t(p, q)$ are located. The natural shape of the input region D_{pq} that enables utilization of all input times is rather trapezium shaped (Fig. 2). However, the method of Fourier coefficients is based on Fourier expansions in the rectangular region $0 \leqq x \leqq x_p$, $0 \leqq q \leqq q_M$, eventually in $0 \leqq x \leqq x_p$, $0 \leqq z(q) \leqq z(q_m)$. This principal disproportion leads to a considerable deterioration of the convergence of the method. A test has shown that the solution converges only in that part of the region D_{xz} delimited by the rectangle constructed on the shorter base of the trapezium D_{pq}. Therefore, the Fourier coefficient method can only be applied if the maximum measurement offset q_M is much smaller than the length of the interpreted interval x_p, i.e., $q_M \ll x_p$.

The Direct Inversion Method

Using the method of Fourier coefficients the velocity distribution is found over a larger region than it is possible to map by means of supporting rays $\Gamma_0(p_j, q_i)$ corresponding to the input time data $t(p_j, q_i)$ available. If the region where the solution is sought were limited strictly to the interpretable region D_{xz} formed by the turning points of seismic rays $\Gamma_0(p_j, q_i)$ (Fig. 2) it could be expected that the linearized theory will bring better results. The grid method presented in this section is one possible way to achieve this. Its principle consists in a sequential ray interpretation of time isolines from the smallest offset to the largest. In contrast to the previous method the interpretation of arrival times is based on the knowledge of surface velocities.

Basic Equations

Let us consider a laterally homogeneous starting model $s_0(z)$ such that the arrival times of the surface-to-surface refracted wave form a continuous function $t_0(q)$ with monotonically decreasing derivative. For this class of starting model it is natural to define the grid points in the plane (p, z) to be the turning points of supporting rays $\Gamma_0(p_j, q_i)$ (Fig. 2). Coordinates of grid lines are then as follows

$$p_j = j\Delta p, \quad j = \ldots, -1, 0, 1, \ldots$$
$$z_i = \xi(q_i), \quad i = 1, 2, \ldots, M,$$

where $\xi(q)$ denotes a continuous and monotonically increasing function of offset q. In individual steps of interpretation we seek the unknown grid values $\Delta s(p_j, z_i)$ on individual grid levels z_i. In the first step we determine only the lateral slowness deviations on the surface using the input values of surface velocities and the mean velocity $s_0^{-1}(0)$ following from the chosen initial model $s_0(z)$. In the successive steps we interpret the individual time isolines $\Delta t(p_j, q_i)$ in the following recursive manner.

Let us assume that we have found the velocity distribution to the depth z_i. Now, we shall divide the integration in expression (5) along the ray trajectory $\Gamma_0(q_{i+1})$ into two parts separated by the grid line $z = z_i$. The integral in the region already mapped for $z \leq z_i$ can be performed. Therefore, we write expression (5) in the form

$$\Delta t(p_j, q_{i+1}) - \int_{0 \leqq z \leqq z_i} \Delta s(x, z)\, d\gamma_0 = \int_{z_i \leqq z \leqq z_{i+1}} \Delta s(x, z)\, d\gamma_0. \tag{13}$$

The left-hand side represents a value of the quantity $\Delta t(p, q)$ related to the subsurface level $z = z_i$. For this we introduce the notation $\Delta t(p_j, q_{i+1}, z_i)$. This quantity can be expressed on the basis of known grid values $\Delta s(p_n, z_m)$, $m = 1, 2, \ldots, i$, in a reasonable numerical approximation. The right-hand side of expression (13) depends on unknown grid values $\Delta s(p_n, z_{i+1})$ on the next grid line $z = z_{i+1}$. If the grid spacing is appropriate to the character of the medium studied, one can restrict oneself to a linear interpolation of function $\Delta s(x, z)$ along this part of the ray path. The right-hand side can then be expressed in linear form with respect to unknown grid values. Expression (13) can then be written in the following form

$$\Delta t(p_j, q_{i+1}, z_i) = C_j + \sum_n D_{jn} \Delta s(p_n, z_{i+1}). \tag{14}$$

Vector C_j as well as matrix D_{jn} are entirely determined by values $\Delta s(p_n, z_i)$ on the preceding grid line $z = z_i$, then by the shape of the ray path $\Gamma_0(q_{i+1})$ in the depth interval $z_i \leqq z \leqq z_{i+1}$ and finally, by the kind of numerical approximation of the line integral. Summation index n goes through the same values as the index j. Matrix D_{jn} is therefore square. It

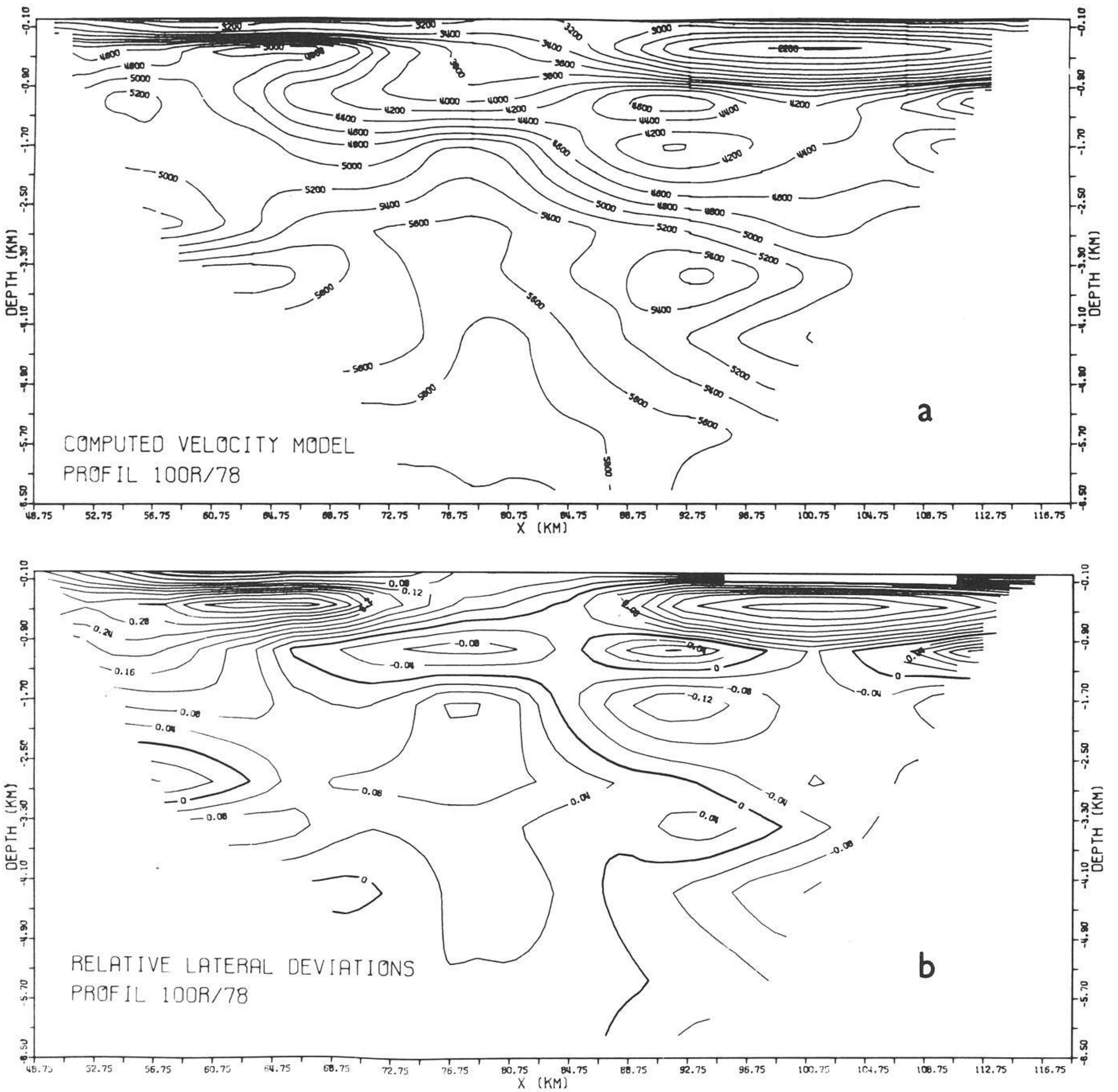

Fig. 7. a Contours of velocity distribution along profile 100R calculated by the method of direct inversion on the basis of selected time isolines according to Fig. 6; **b** Contours of relative lateral deviation between the starting model $s_0(z)$ and the calculated one $s(x, z)$

can be shown that it is symmetrical with a limited number of non-zero diagonals given by the number of grid values $\Delta s(p_n, z_{i+1})$ on line $z = z_{i+1}$ necessary to perform the interpolation of $\Delta s(x, z)$ along the deepest part of the ray $\Gamma_0(p_j, q_{i+1})$. In particular, if the interpolation in the lateral direction is suppressed, a diagonal form of matrix A_{jn} is obtained. In this case, one input time value $t(p_j, q_{i+1})$ contained at the left-hand side of the relation (14) results in one grid slowness value $\Delta s(p_j, z_{i+1})$. This property of direct inversion $\Delta t(p_j, q_{i+1}) \rightarrow \Delta s(p_j, z_{i+1})$ led to the name of the method.

The system of linear Eq. (14) thus enables us to state all unknown grid values $\Delta s(p_n, z_{i+1})$ on the basis of the known time differences $\Delta t(p_j, q_{i+1})$ of the interpreted $(i+1)$-th time isoline. After performing all M steps of the interpretation, the entire region D_{xz} is then mapped out. Expression (14) is the basic one for the method of direct inversion.

Applications

On the basis of expression (14) a numerical algorithm was derived and a program implementing the method of direct inversion was written. Input data are surface velocities and arrival times of surface-to-surface refracted waves arranged according to individual measurement offsets in the interval investigated $\langle 0, x_p \rangle$. The accuracy of the line integral calculation that occurs in expression (13) depends on the subdivision of the ray path $\Gamma_0(q)$. In the computer implementation the accuracy of a calculation of C_j coefficients as well

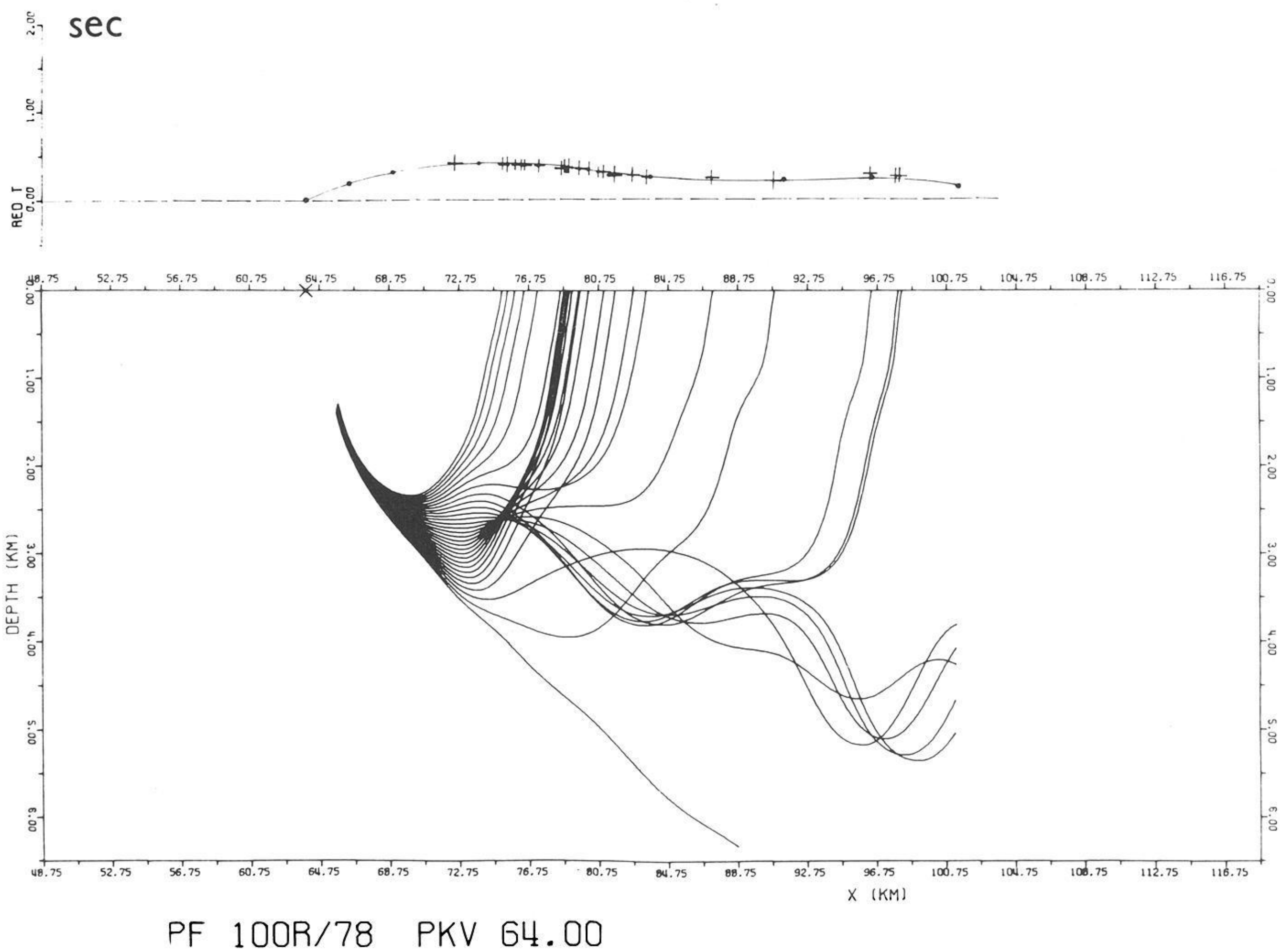

Fig. 8. Test of the calculated velocity model in Fig. 7a for the position of the source $x_S = 64.00$ km. The reduction velocity is 5 km/s. ···, arrival times derived from input time isolines; + + +, arrival times calculated by means of a ray tracing program. The corresponding ray paths are illustrated in the lower part of the figure

as matrix elements D_{jn} is controlled by choice of an adequate number of subelements of the ray path between neighbouring grid lines z. Further, the calculation of the velocity distribution can be performed at any sub-interval of the basic interval $\langle 0, x_p \rangle$.

The supporting travel-time curve $t_0(q)$ is obtained by centering the individual time isolines within the chosen sub-interval. The starting model $s_0(z)$ is then derived by the Wiechert-Herglotz transformation of a smoothed version of $t_0(q)$. In this way we always interpret the minimum lateral time deviations in the chosen sub-interval, when the linearized theory works best (Fig. 1). By defining sub-intervals with certain overlapping parts and by a mutual comparison of the velocity models calculated here, the consistency of the linearized approach can also be verified.

The method has been tested on theoretical as well as real data. Figure 5 shows the result of an interpretation of theoretical time isolines corresponding to a velocity model with constant velocity gradients. The lateral inhomogeneity reached nearly $\pm 50\%$ in the upper part of the section (Fig. 5b). In spite of this fact, isolines of the calculated velocity distribution are in good agreement with the theoretical equidistant straight lines over the entire region of the solution obtained, with the exception of a narrow region close to the margins (Fig. 5a). A quantitative evaluation of the method's accuracy is presented in Fig. 4. Curve (3) shows the dependence of relative accuracy $\eta(z)$ on the depth z according to expression (12). In comparison with the one-step method of Fourier coefficients (curves 1 and 2) the method of direct inversion shows a substantially better accuracy.

Time isolines in Fig. 6 were drawn from refraction data recorded along a part of the Carpathian profile 100 R in West Slovakia (Rektořík et al. 1979). In this portion of the profile time isolines exhibit a considerable lateral dependence corresponding to a transition from the region of pre-Neogene formations into Neogene basins. The contour map of the velocity distribution in Fig. 7 was derived from calculated grid data $\Delta s(x, z)$ as well as contour lines of the relative lateral deviation from the velocity starting model. It is obvious from this figure that the lateral deviation varies from approximately $+40\%$ in the region of pre-Neogene formations to -30% in the region of lower velocities in the Neogene sedimentary basins. To verify the results we chose the two longest travel-time curves with shot points fixed at $x = 64.0$ km and 93.9 km (solid lines in Figs. 8 and 9). By means of a ray tracing program (Červený et al. 1977) the direct problem for the obtained velocity model was solved. The ray paths, together with corresponding travel times for the shot points tested, are illustrated in Figs. 8 and 9. In spite of the complex character of the model tested (strongly variable gra-

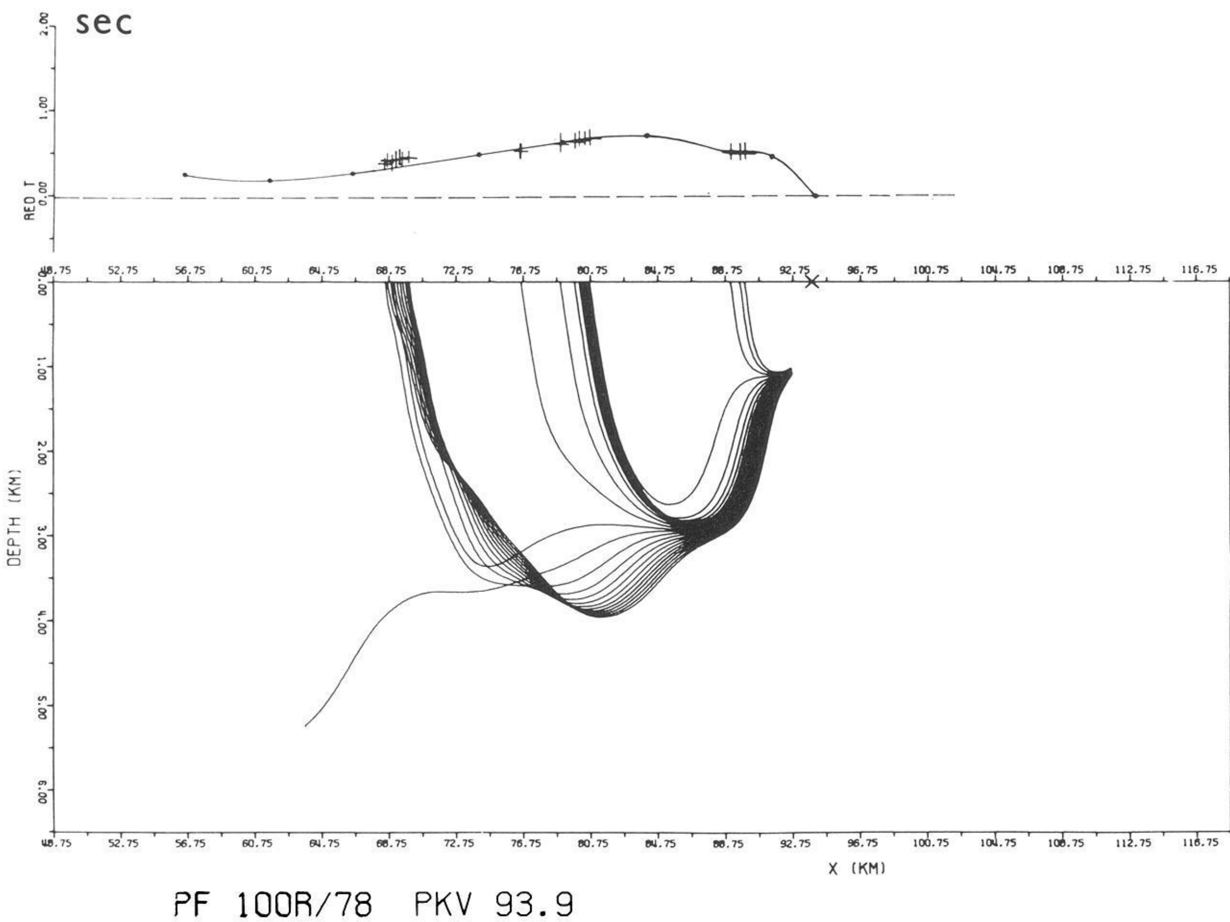

Fig. 9. Test of the calculated velocity model for the source position $x_S = 93.90$ km. The reduction velocity is 5 km/s. + + +, arrival times calculated by means of ray tracing program. The corresponding ray paths are illustrated in the lower part of the figure; ..., arrival times derived from input time isolines

dients, local velocity inversions) time discrepancies larger than 2% have not been detected in the tests.

Recently the method has been applied on further regional refraction profiles in Czechoslovakia (Novotný et al. 1981).

Concluding Remarks

Methods for solving inverse seismic problems can be divided into two groups:

(i) Evolutional, multi-step methods that, on the basis of surface data, map the seismic medium successively from the surface down to some depth. The result of each step is dependent on results of preceding steps. Included here, for instance, are the so called wave-equation migration methods which form an image of the seismic medium using information carried by the reflected wave field (Claerbout 1971). Of the methods for solving the inverse kinematic problem mentioned above, the differential method (Belonosova et al. 1967) and the method of characteristics (Romanov et al. 1978) can be designated as evolutional, as can the method of direct inversion presented in the section above.

(ii) One-step methods where the solution is sought for the entire region investigated by means of an optimization procedure. The method suggested by Romanov (1972) is a one-step methods as is the method of Fourier coefficients derived above.

As far as the character of the problem allows, it is advantageous to use numerical algorithms of the evolutional type (lower requirements for computer memory, mostly simpler algorithms, a natural possibility of intermediate results management). In spite of various numerical implementations, common principles of evolutional algorithms can be found, i.e.,

- the values of the interpreted quantity registered at the surface ($z=0$) are recursively transformed to lower and lower depth levels $0 < z_1 < z_2 < z_3 \ldots$,
- the result of the interpretation is yielded in terms of the quantity transformed for the zero travel time of the seismic wave of interest.

In the case of holoseismic techniques the reflected wave field is transformed. While solving the inverse kinematic problems, we transform the travel time of refracted waves directly. In particular, the transformation of time isolines used by the direct inversion method consists in the recalculation of the travel-times $t(R, S)$ from the position of source S and receiver R on the surface $z=0$ recursively to lower and lower depth levels $z=z_i$. In the limit, when, during the recalculation, coincidence of the source and receiver is achieved (i.e., transformed travel-time reaches zero), the value of the slowness function can be obtained by differentiating $t(R, S)$ with respect to the distance q of the source and receiver. In terms of p, q introduced in Eq. (1) this transformation principle assumes the following simple form

$$\Delta s(x, z) = \left.\frac{\partial t(p, q, z)}{\partial q}\right|_{\substack{p=x \\ q \to 0}} \tag{15}$$

where $t(p, q, z)$ is the travel time recalculated along the ray path $\Gamma(p, q)$ to the level z. An expression like (15) in a finite-difference form represents the basic equation of the method suggested by Belonosova et al. (1967). To extrapolate the surface arrival times depth, a non-linear differential equation is solved by finite-difference approximation.

In the linearized approach used in the direct inversion method, the extrapolation is realized in a form of a line integral, such as the left-hand side of expression (13) for the deviation $\Delta t(p, q, z)$ of the propagation time. The right-hand side of expression (13) then represents the linearized analogue of the transformation principle (15) in integral form: the limiting transition $q \to 0$ is replaced by the linear interpolation of the function Δs along the ray path trajectory. It appears that the choice of the natural grid points as the vertices of the supporting seismic ray paths where the relation (15) is fulfilled, in particular, leads to a high accuracy for the linearized method even in the case of a low density of grid points.

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