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An Improved Algorithm for Magnetotelluric and Direct Current Data Interpretation

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Abstract. A simple iterative inversion method for MT and DC data, which is a significant improvement of the usual automatic trial and error process, is presented. The basic idea has long been used, i.e. inversion using approximate solution and exact forward computation for check of fit. In the new version the iterations are performed in the resistivity vs. depth domain instead of the apparent resistivity vs. frequency in MT or apparent resistivity vs. electrode separation in DC soundings. This modification totally avoids the appearance of the physically meaningless solutions which are a major obstacle in the common iterative procedure. Thus a sufficiently good fit is always reached after only a few iterations. As in all similar inversion methods, a solution consists of as many layers as the number of sampled points. The number of layers may be reduced by various methods. A simple interactive method for the reduction of number of layers, which was found very useful, is presented. It is based on an approximate solution which enables the simultaneous creation of a wide range of equivalent sections from the original multilayered section derived in the inversion process. The new inversion and reduction of layers procedures can be easily programmed. They have been applied to synthetic as well as on numerous field soundings and were found to be powerful in the interpretation of field surveys.

Key words: Inversion – Magnetotellurics – Direct current resistivity soundings

Introduction

A quantitative interpretation of magnetotelluric and direct current field data is still the most problematic and critical step in magnetotelluric (MT) and DC resistivity surveys. Since two and three dimensional interpretation is not routinely possible for practical reasons, a one-dimensional interpretation is the most which may be expected from the exploration geophysicists who handle a large number of soundings.

Anyone who has tried to carry out an interpretation based on a purely automatic process will probably agree that such interpretation is often very far from being conclusive and realistic. The reason for the discrepancy between true resistivities and the derived one, apart from inhomogeneity, is in the existence of a wide range of equivalent solutions which are indistinguishable for geoelectrical methods. Thus it seems reasonable to derive some basic multilayered section which fits the field data sufficiently well and then to create a wide range of equivalent sections from which the most suitable section may be chosen (with the aid of additional geological data).

Two main automatic inversion methods for MT and DC data are widely used. One of them is based on least mean square fitting technique (Wu 1968; Patrick et al. 1969; Inman et al. 1973). The other one is an iterative method based on approximate (asymptotic) inversion (Maillet 1947; Berdichevsky 1965; Bostick 1977) and an exact forward computation for comparison of the solution with the data (Zohdy 1973, 1975; Chaipayungpun and Landisman 1977).

The least mean square technique is based on the generalized linear inverse theory. This method requires the exhaustive computation of all the eigenvalues and eigenvectors of the system matrix. By introducing logarithmic variables, controlling the eigenvalue cut-off and looking at the eigenvectors, equivalences are detected.

The second method seems to be very attractive since it fits exactly the given set of data without any need for an initial guess. The disadvantage of this method, when used in the usual way, is in the appearance of physically meaningless solutions such those that include negative resistivities or negative thicknesses. The modification of the original scheme, made in order to avoid this problem (Zohdy 1973; Chaipayungpun and Landisman 1977), usually results in loss of accuracy.

An improved algorithm, suitable for the inversion of MT apparent resistivity and DC Kernel function curves, is presented. This algorithm is absolutely free from physically meaningless resistivities and thicknesses and always leads to a high degree of accuracy. The algorithm is based on fitting of resistivity-depth curves, which are a product of asymptotic inversion, instead of apparent resistivity curves.

The number of layers derived in the inversion process is equal to the number of sampled points. In order to get a useful solution, the number of layers must be reduced. This may be done by an automatic method (Zohdy 1975), but a simple and more powerful technique is presented here. This technique, also based on the asymptotic equations, allows not only a reduction in the number of layers, but also the simultaneous creation of a wide range of equivalent sections without significant loss of accuracy. The use of this technique permits the inclusion of all available geological and geophysical data, thus leading to the best possible subsurface section.

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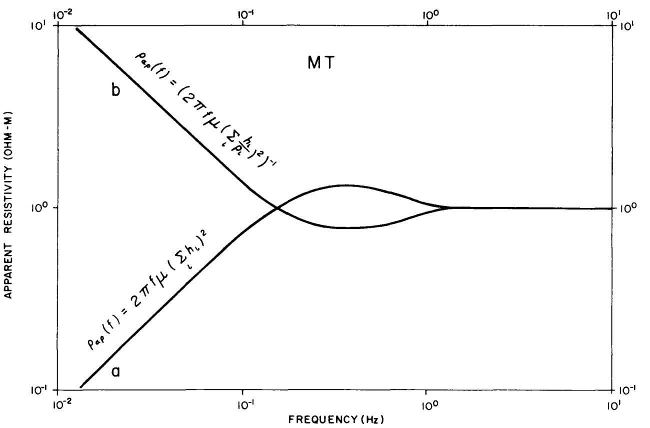

Fig. 1. MT apparent resistivity curves computed for a single layer (1 ohm-m) 100 m thick, underlayed by an ideal conducting (curve *a*) and an ideal insulating (curve *b*) half space

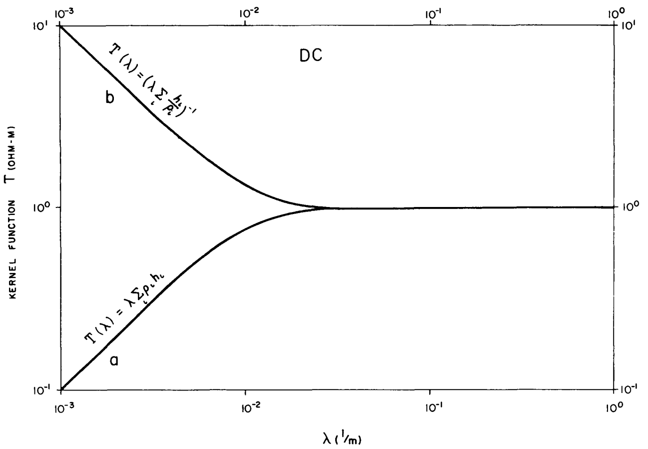

Fig. 2. DC kernel function computed for a single layer (1 ohm-m), 100 m thick, underlayed by an ideal conducting (curve *a*) and an ideal insulating (curve *b*) half space

The algorithms for inversion and for reduction of the number of layers were applied to a large number of synthetic and field examples. Our experience shows that the interpretation procedure which is described in this paper is very useful due to simplicity, the consistent, meaningful results and its almost negligible cost.

Approximate (Asymptotic) Expressions for Inversion of MT and DC Curves

The idea of deriving an approximate solution by computing apparent resistivity as a point of intersection of two low frequency asymptotes, was originally introduced by Bostick (1977) for an inversion of MT data, assuming a continuous change of resistivity with depth. Using the same method, similar asymptotic solutions for discrete changes of resistivity may be derived. The same approach may be extended to the case of DC and thus the nature of the expressions used by Zohdy (1973, 1975) for the inversion of Schlumberger apparent resistivity curves can be clarified.

Figures 1 and 2 illustrate MT apparent resistivity and DC Kernel function curves for a single layer ($\rho=1.0$ ohm-m, $h=100$ m) overlaying a perfect conductor (curve *a*) or a perfect insulator (curve *b*). One can see that the apparent resistivity (Kernel function for DC) values at frequencies higher than the frequency of intersection for both low frequency asymptotes are only slightly affected by an infinite change in resistivity of the half-space. Furthermore, at the frequency of the asymptotes intersection, the corresponding apparent resistivity (Kernel function) values appear to be a reasonable single-value, central estimate of the small range of values, over which the exact apparent resistivity varies in response to all possible changes in the underlying half-space conductivity. Thus it may be assumed that it is almost independent of underlying structure as well. Therefore, at any frequency the apparent resistivity value can be approximately presented as a common value of both low frequency asymptotes.

Starting with a well known recursive formula for direct computation of electromagnetic impedance over a one dimensional layered earth (Word et al. 1970), asymptotic expressions for apparent resistivity (ρ_{ap}) are derived:

$\lim\limits_{\omega\to 0} \rho_{ap}/\omega=\mu\left(\sum\limits_{j=1}^{N-1} h_j\right)^2$, for a case of conducting half space ($\rho_N\to 0$)

and $\lim\limits_{\omega\to 0} \omega\rho_{ap}=1\Big/\left[\mu\left(\sum\limits_{j=1}^{N-1} h_j/\rho_j\right)^2\right]$, for a layered medium overlaying a perfect insulator ($\rho_N\to\infty$).

As usual, ω denotes the frequency μ is the permeability, j is the layer numbered from the top and N is the number of layers.

Combining those asymptotic expressions, we get a system of equations which make possible an approximate computation of MT apparent resistivity in terms of sub-surface parameters:

$$\begin{aligned}\rho_{ap}(\omega_n)&=\mu\omega\left(\sum_{j=1}^{n} h_j\right)^2\\ \rho_{ap}(\omega_n)&=1\Big/\left[\mu\omega\left(\sum_{j=1}^{n} h_j/\rho_j\right)^2\right]\end{aligned}\tag{1}$$

where n is the deepest layer (it may be a fraction of a layer as well) which still affects th apparent resistivity at frequency ω_N.

Slightly modifying and introducing $D_n=\sum\limits_{j=1}^{n} h_j$ – the depth to the $(n+1)$-th layer:

$$\begin{aligned}D_n&=(\rho_{ap}(\omega_n)/\mu\omega_n)^{1/2}\\ \rho_{ap}(\omega_n)&=D_n\Big/\left(\sum_{j=1}^{n} h_j/\rho_j\right).\end{aligned}\tag{2}$$

Equation (2) was originally derived by Jain (1966) using simple physical concepts. The expression found here for D_n is just $1/\sqrt{2}$ of the homogeneous earth skin depth and is close to two-thirds of the skin depth experimentally found and used by Jain.

Since $\sum\limits_{j=1}^{n-1} h_j/\rho_j=D_{n-1}/\rho_{ap}(\omega_{n-1})$ and $h_n=D_n-D_{n-1}$, convenient expressions for approximate inversion of MT apparent resistivity curves can be derived from Eq. (2) (a decreasing order of frequencies is assumed):

$$\begin{aligned}\rho_n&=h_n\Big/\left(\frac{1}{\sqrt{\rho_{ap}(\omega_n)\,\omega_n\mu}}-\frac{1}{\sqrt{\rho_{ap}(\omega_{n-1})\,\omega_{n-1}\mu}}\right)\\ h_n&=\frac{\rho_{ap}(\omega_n)}{\omega_n\mu}-\frac{\rho_{ap}(\omega_{n-1})}{\omega_{n-1}\mu},\qquad n=2,\dots,N.\end{aligned}\tag{3}$$

$\rho_1=\rho_{ap}(\omega_1)$ and $h_1=\sqrt{\rho_{ap}(\omega_1)/\mu\omega_1}$ are obtained from (1). h_N, being the thickness of a half space, is meaningless, and is required only for the computation of the last resistivity ρ_N.

The number of layers derived by this method is equal to the number of sampled points. This does not mean that it is the actual number of layers since such a layer may be only a fraction of a real geological layer.

The depth to the n-th layer depends on $\rho_{ap}(\omega_n)$ and ω_n only. Therefore, it is possible to invert independently separate parts of the apparent resistivity curves. This property is most significant when, due to a poor signal to noise ratio, the apparent resistivity data is not continuous over a wide frequency band, but rather consists of a few discrete data segments.

Assuming a continuous change of resistivity with depth and using the same asymptotic expressions, it is possible to derive a similar set of equations for MT apparent resistivity data inversion (Bostick 1977):

$$D=\sqrt{\frac{\rho_{ap}(\omega)}{\omega\mu}},$$

$$\rho=\rho_{ap}(\omega)\frac{1-d(\log\rho_{ap}(\omega)/d(\log\omega)}{1+d(\log\rho_{ap}(\omega)/d(\log\omega)}.$$

The asymptotic expressions for DC Kernel function are derived from direct computation recursive formula:

$\lim_{\lambda\to 0} T/\lambda=\sum_{j=1}^{N-1}\rho_j h_j$ for a layered medium overlaying a conducting half space ($\rho_N\to 0$) and

$\lim_{\lambda\to 0}\lambda T=1\Big/\left(\sum_{j=1}^{N-1}h_j/\rho_j\right)$ for a layered medium overlaying a perfect insulator ($\rho_N\to\infty$).

Combining the asymptotic expressions we get a system of equations which makes possible an approximate computation of DC Kernel function values in terms of sub-surface parameters:

$$T(\lambda_n)=\lambda_n\sum_{j=1}^{n}\rho_j h_j$$
$$T(\lambda_n)=1\Big/\left(\lambda_n\sum_{j=1}^{n}h_j/\rho_j\right). \tag{4}$$

From Eq. (4), using $\sum_{j=1}^{n-1}\rho_j h_j=T(\lambda_{n-1})/\lambda_{n-1}$ and $\sum_{j=1}^{n-1}h_j/\rho_j=1/(T(\lambda_{n-1})\cdot\lambda_{n-1})$, convenient expressions for the inversion of DC Kernel function curves are derived:

$$h_n=\sqrt{\left(\frac{T(\lambda_n)}{\lambda_n}-\frac{T(\lambda_{n-1})}{\lambda_{n-1}}\right)\left(\frac{1}{\lambda_n T(\lambda_n)}-\frac{1}{\lambda_{n-1}T(\lambda_{n-1})}\right)}$$

$$\rho_n=\sqrt{\left(\frac{T(\lambda_n)}{\lambda_n}-\frac{T(\lambda_{n-1})}{\lambda_{n-1}}\right)\Big/\left(\frac{1}{\lambda_n T(\lambda_n)}-\frac{1}{\lambda_{n-1}T(\lambda_{n-1})}\right)}$$

$\rho_1=T_1$ and $h_1=1/\lambda_1$ are obtained from (4).

Similar expressions were used by Zohdy (1975) in the inversion of Schlumberger DC apparent resistivity curves (but with apparent resisitivity instead of Kernel function). The slopes of the Kernel function are limited by ± 1, whereas the slopes of DC apparent resistivity curves may exceed -1. Thus the use of expression (5), which is well suiled to kernel curve inversion, in the inversion of apparent resistivity curves, requires certain modifications (Zohdy 1973, 1975).

Inversion of MT Apparent Resistivity and DC Kernel Curves by an Iterative Process

The expressions for MT and DC curve inversion derived in the previous section represent a low frequency approximation. There will be an inherent discrepancy between the original curves and the curves computed by an exact forward computation for the approximately derived resistivity section. Furthermore, the approximate multi-layered section includes almost gradual changes of resistivity instead of well defined interfaces.

It was shown by Zohdy (1973, 1975) for DC, and by Chaipayungpun and Landisman (1977) for MT, that improving the accuracy of the asymptotic method is possible by an iterative process based on the same asymptotic expressions. The most serious limitation of these proposed methods is that they produce slopes which exceed the range of -1 to $+1$ and thus lead to meaningless solutions. The modifications of the original iterative process scheme, introduced by Zohdy (1973, 1975) and Chaipayungpun and Landisman (1977) in order to avoid the above mentioned problem, may result in a significant loss of accuracy or an inability to converge.

In an effort to avoid unreasonable slopes and the problems which they create, a new algorithm has been developed. The proposed algorithm is based on the existence of a one-to-one correspondence between the apparent resistivity (MT) and Kernel function (DC) curves or resistivity sections and the corresponding approximate resistivity sections derived using the asymptotic solutions. This fact enables us to compare and fit these resistivity sections instead of apparent resistivity curves.

Since approximate resistivity sections for DC soundings are computed from Kernel function values, the low resolving power of Kernel function curves for a C-type ($\rho_1>\rho_2>\rho_3$) and certain HK-type ($\rho_1>\rho_2<\rho_3>\rho_4$) sections is inherent in corresponding approximate resistivity sections and may result in a final solution which fits the Kernel function curve well but does not necessarily fit the apparent resistivity curve. Therefore, as in interpretation in the Kernel domain, the calculation of apparent resistivity curves for comparison with the field data is advisable, especially for curves with steeply descending branches (Zohdy 1975).

Starting with any section, usually with an asymptotic solution for the given apparent resistivity curve, we shall compare the approximate sections corresponding to this particular section and to the given apparent resistivity curve. We assume that the disagreement existing between the approximate sections is close to the disagreement existing between the exact solutions.

Thus, we are able to change, step by step, the previous guess toward the exact solution. Resistivity and thickness values are compared and modified, using a logarithmic scale, so that negative values are avoided. The different steps for the proposed method are as follows:

1. From the given apparent resistivity curve $\rho^m_{ap}(\omega_i)$, $i=1,2,\ldots,n$, where i is the index number of each sample, using asymptotic expressions, compute the corresponding resistivity section ρ^m_i, h^m_i, $i=1,2,\ldots,n$. In order to preserve the one-to-one correspondence existing between the samples of apparent resistivity and the segments of the resistivity section, the samples must be arranged from high frequencies towards the lower ones, if the layers are counted downwards.

2. Use ρ^m_i, h^m_i, $i=1,2,\ldots,n$ as ρ_i, h_i, $i=1,2,\ldots,n$ – a first approximation of the true layering.

3. From ρ_i, h_i, $i=1,2,\ldots,n$ compute $\rho^c_{ap}(\omega_i)$, $i=1,2,\ldots,n$ by exact forward computation.

4. Compare $\rho^c_{ap}(\omega)$ with the given curve $\rho^m_{ap}(\omega)$ at all ω_i; if the requested fit is achieved then ρ_i, h_i, $i=1,2,\ldots,n$ is the requested solution.

5. If significant disagreement between ρ^c_{ap} and ρ^m_{ap} still exists, from $\rho^c_{ap}(\omega)$, using asymptotic inversion, compute the corresponding resistivity section ρ^c_i, h^c_i, $i=1,2,\ldots,n$.

6. Change the values of ρ_i and h_i according to the disagreement between ρ^m_i and ρ^c_i, h^m_i and h^c_i, i.e. replace $\log\rho_i$ by $(\log\rho_i+\log\rho^m_i-\log\rho^c_i)$ and $\log h_i$ by $(\log h_i+\log h^m_i-\log h^c_i)$ for all and use the new values in the next iteraction. Return to step 3.

All the stages are valid for DC Kernel curve inversion if $\rho_{ap}(\omega)$ is changed to $T(\lambda)$.

Inversion of MT Data

The proposed method of iterative inversion was applied to a large number of synthetic and field data. The synthetic apparent resistivity curves were computed using the usual forward computation technique (Word et al. 1970). The iterative inversion, when used in all practical cases, was found to be very fast (3 to 5 iterations to get an accuracy better than 3–5%), successful and sufficiently accurate. However, in order to demonstrate the features of this method in the present discussion we shall look for the usually unreasonable relative accuracy of 10^{-6} and we shall limit the maximum number of iterations to 300. As will be illustrated, the significant improvement occurs after just 3 to 5 iterations. Higher number of iterations, even if improving the fit, usually lead to geologically meaningless solutions.

A typical example is presented in Figure 3. When two samples per decade are used, the convergence is very fast and the requested accuracy (10^{-6}) is usually achieved after only 20–40 iterations. A problem arises, however, that except for the sampled points, the accuracy was only slightly improved when compared to the results of the direct asymptotic inversion (Fig. 3b). When the apparent resistivity curve is sampled at four points per decade, the accuracy is improved over all of the curve (Fig. 3c), but the convergence becomes very slow and the requested fit (10^{-6}) was not achieved even after 300 iterations.

In Figure 4 the corresponding resistivity – depth cross-sections are presented (wavy line at the bottom of the resistivity profile represents the transition to the homogeneous half space): derived without iterations (a); after three iterations (b); after 300 iterations (c). The apparent resistivity curve in this example was sampled at four points over a decade.

The solution reached after three iterations (Fig. 4b) seems to be the one most similar to the given subsurface cross-section. The solution derived after 300 iterations (Fig. 4c), which may be expected to be the best, since the corresponding apparent resisitivity curve is the closest to the one given , is actually the worst. Such a correlation indicates that a good fit, with an accuracy much better than that usually available in field data, does not guarantee that the solution reached is the real one rather than one of a wide range of equivalent solutions. This fact illustrates the limited resolution of the magnetotelluric method: very simplified solution, like the sections used in this example, may be found in case of a very complicated geology, similar to the solution found after 300 iterations.

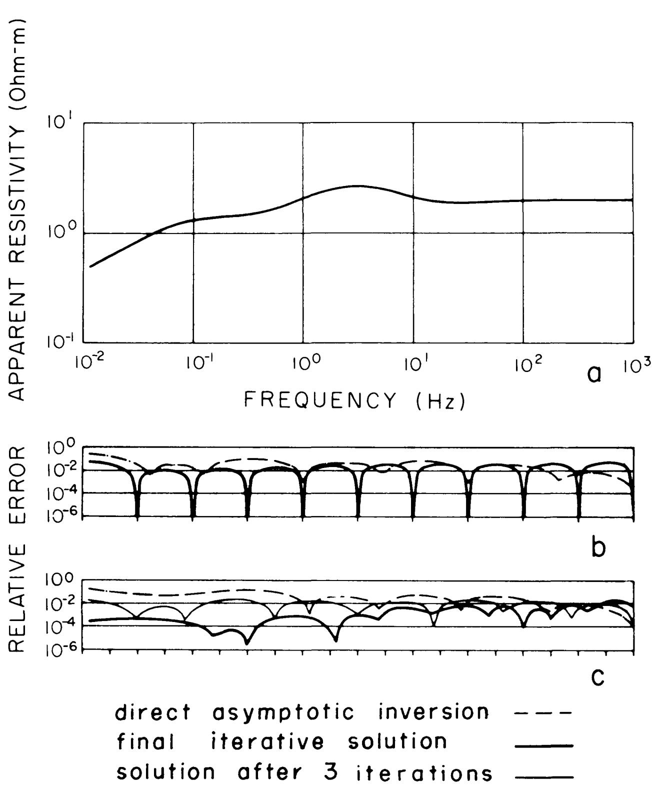

Fig. 3a–c. Inversion of synthetic model – error analysis (MT): **a** apparent resistivity curve (5 layers); **b** relative error for rough sampling (2 points per decade); the iterative process was terminated when the requested accuracy was reached at in the sampled points: **c** relative error for increased sampling (4 points per decade); the iterative process was terminated after 300 iterations

For practical purposes, it is customary to derive a simple solution as a geological model. With this in mind, the solution derived after only a few iterations seems to be sufficient in the sense that both the apparent resistivity and the resistivity sections are similar to the synthetic ones.

The solution derived using this inversion method consists of as many layers as there are sampled points. In the next section, a method of interpretation based on the reduction of the number of layers is presented.

Reduction of a Number of Layers – a Practical Approach to the Interpretation of MT Data

A product of 3–5 iterations, such as those presented in Figure 4b with all of the disadvantages mentioned, usually cannot be used as a final solution. In its present form the solution consists of a large number of layers while a geological interpretation based on several major lithological layers is usually needed. Thus, the next step must be a reduction of the number of layers with a minimal loss of accuracy achieved in fitting the apparent resistivity curves.

The technique presented in this section is a very simple one and is based on the same asymptotic expressions used in the inversion process. This method was found to be a very effective tool in interpretation, since it makes possible the inclusion of all the known subsurface information from the surveyed area.

Let us examine the section derived using the proposed iterative inversion method. Instead of only a few thick homogeneous and geologically justifiable layers, this section con-

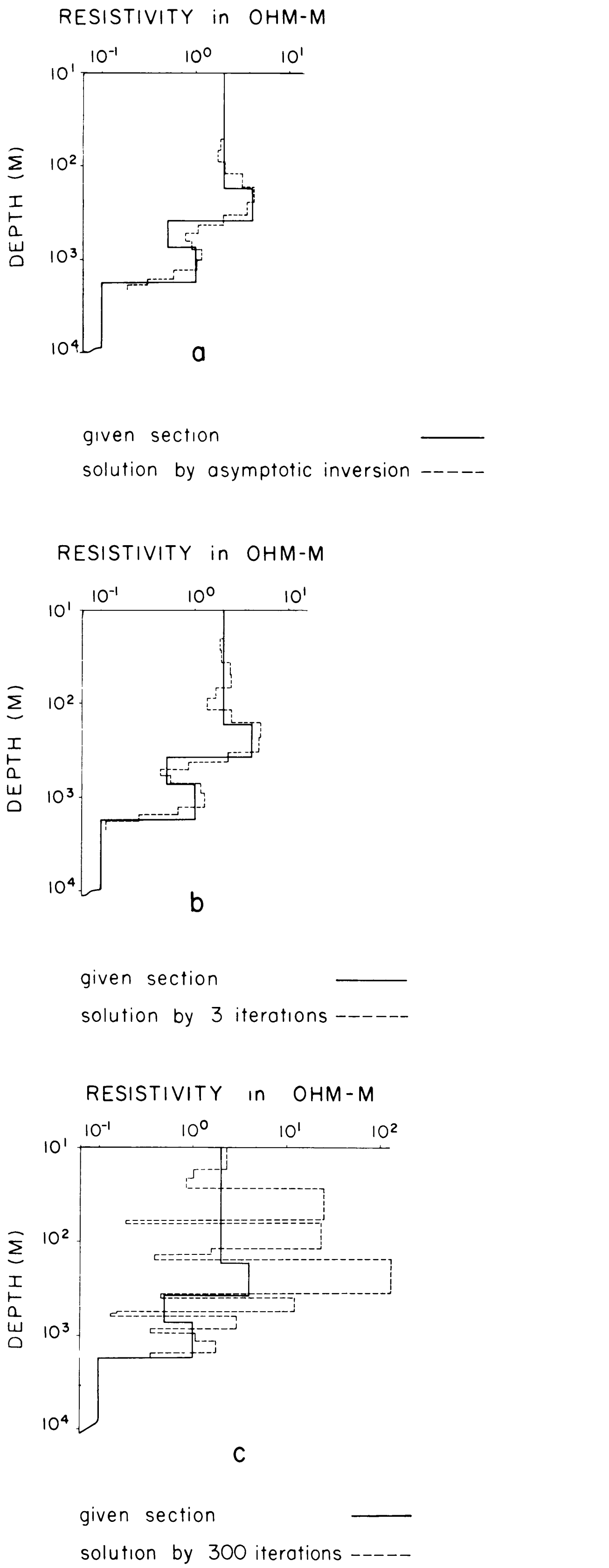

Fig. 4a–c. Subsurface resistivity structure derived for synthetic model (shallow layers are omitted): **a** by direct asymptotic inversion; **b** by 3 iterations; **c** by 300 iterations

sists of a large number of layers with an almost gradual change of resistivity from one layer to another (Fig. 4b). If it is certain that two layers (if no geological data is available, layers with extreme resistivities may be used) k and l belong to two neighbouring geological layers, it is necessary to divide and to add (in a certain way) the interlying pseudolayers $k+1$, $k+2$, ..., $l-1$ to the layer k and l. Thus, an equivalent two-layers solution with resistivities ρ_k and ρ_l and corresponding thicknesses $h_k + \Delta h_k$ and $h_l + \Delta h_l$ will be found for this part of the multilayered section.

If the resistivities of the two-layer section are defined a priori, the choice of Δh_k and Δh_l must be made in such a way that the corresponding apparent resistivity curve will remain similar to the one corresponding to the original multilayered section.

It is clear that if

$$\begin{aligned} &\Delta h_k + \Delta h_l = \sum_{j=k+1}^{l-1} h_j \\ &\frac{\Delta h_k}{\rho_k} + \frac{\Delta h_l}{\rho_l} = \sum_{j=1}^{l-1} \frac{h_j}{\rho_j}, \end{aligned} \tag{6}$$

the apparent resistivity values, computed using approximate forward computation expressions (1), will not change at frequencies $\omega_1, \ldots, \omega_k, \ldots, \omega_n$ and the curve will be smoothed only between ω_k and ω_l. It was found that even the exact computed apparent resistivity curves remain almost unchanged if equalities (6) hold. Consequently, dividing the multilayered section, using this method, a geologically meaningful equivalent section can be obtained. If the existence of any layer with a given resistivity is known, and it does not appear in the resistivity section produced by the automatic inversion process, it can be added using a zero thickness at a suitable depth; through Eq. (6), its interfaces with the neighbouring layers can then be defined.

Sometimes, negative values for $h_k + \Delta h_k$ or $h_l + \Delta h_l$ may be reached. Usually, this happens only when an unsuitable choice of k and l is made.

Finally, when the requested number of layers is reached, it is necessary to compute the corresponding apparent resistivity curve and compare it with the given one in order to guarantee that the final solution remains satisfactory.

In Table 1 the possibility for layer number reduction based on Eq. (6) in the case of a synthetic model is illustrated. At the first step, a 20-layered section presented in Fig. 4b was derived by the iterative method. The iterative process was terminated after three iterations. The relative error reached in corresponding apparent resistivity curve fitting is presented in Figure 3c; the errors are less than 2.8% (without iterations −16.6%). Using the method described, the number of layers was reduced to five. Extremely high and low resistivity values were chosen to represent the actual structure. As seen from Table 4 the derived section is close to the synthetic one. The fit of the apparent resistivity curve for the final solution is better than 4.4%.

A field example is presented in Fig. 5. A good fit of both apparent resistivity and phase data is evidently reached by the proposed method.

Inversion and Interpretation of MT Apparent Resistivity Curves – Summary

As shown above relatively good accuracy may be reached by the proposed iterative method after only a few iterations. This

Table 1. Original resistivity model and a resistivity depth section derived from the computed MT apparent resistivity values using the proposed inversion routine and layer number reduction scheme

Layer No.		1	2	3	4	5
Synthetic model	ρ (ohm-m)	2.00	4.00	0.50	1.00	0.10
	h (m)	170.00	300.00	250.00	1,000.00	–
Derived section	ρ (ohm-m)	2.01	5.04	0.43	1.28	0.12
	h (m)	176.78	294.96	211.26	993.28	–

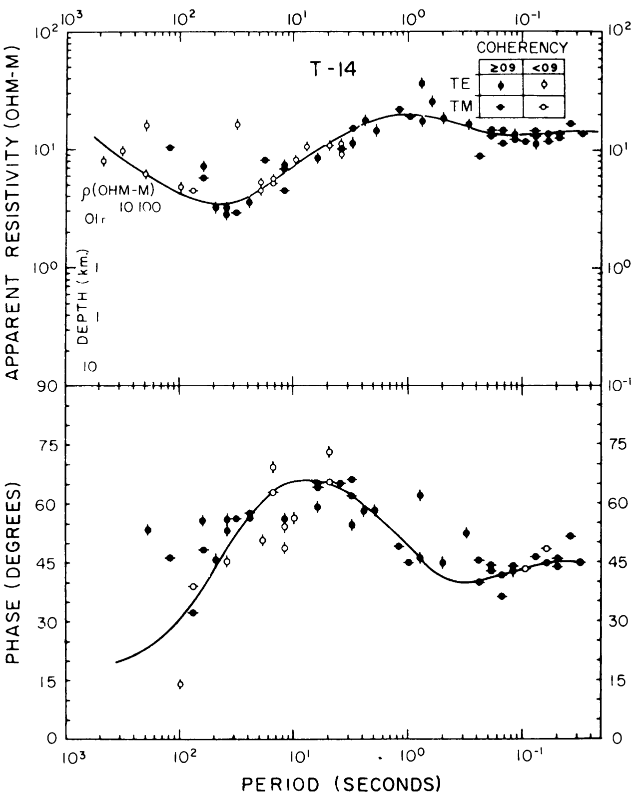

Fig. 5. Field example for MT apparent resistivity curve fitting

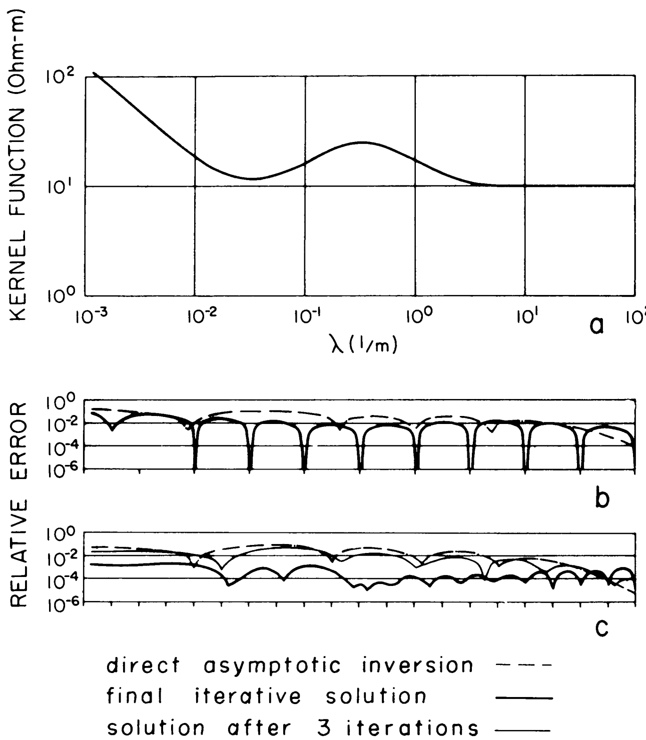

Fig. 6a–c. Inversion of synthetic model – error analysis (DC): **a** apparent resistivity curve (5 layers); **b** relative error for rough sampling (2 points per decade); the iterative process was terminated when the requested accuracy was reached at the sampling points; **c** relative error for increased sampling (4 points per decade); the iterative process was terminated after 300 iterations

accuracy is only slightly affected by the layer number reduction. Thus, the different stages of the proposed inversion and interpretation process may be summarized as follows:

(a) Sampling of the apparent resistivity curve, 3–4 samples per decade (not necessarily uniformly spaced); if the data is noisy, the phase values may be taken into account in order to define the general shape of the noisy segments more clearly[1].

Since the inversion process does not use phases, the resulting phase curve in the presence of noise may be inconsistent with the field measured data even when sufficient accuracy of apparent resistivity is reached.

(b) Inversion of the sampled apparent resistivity curve using the iterative process; from experience, reasonable values for accuracy requested are 3 to 6%. While trying to improve accuracy, the number of iterations will essentially increase, thus leading to very complicated sections which do not allow any reasonable geological interpretation. For this same reason, the number of iterations must be limited to 7–10 (it was found that the most significant improvement in the fitting of apparent resistivity curves occur on just the first few iterations). The requested accuracy may, possibly, not be reached at several samples. Usually, this occurs when dealing with very noisy data (it does not happen at all with synthetic data).

In such a case it would be advantageous to compare the accuracy reached with the data quality.

(c) Interpretation of the solution using the layer number reduction technique. All available geological information must be used at this step in order to obtain a reasonable solution. The interpretation must be made carefully and the limited resolution of the MT method must always be considered.

(d) Finally, it is necessary to compute the apparent resistivity curve corresponding to the final solution in order to make sure that the derived geological section is consistent with the measured data.

Inversion of DC Data

The proposed method of iterative inversion was applied to a large number of synthetic and field measured DC data.

In the following example (Fig. 6), in order to demonstrate the inversion techniques, the iterative process was terminated when accuracy better than 10^{-6} was reached or after 300 iterations. However, in all practical cases the field data quality is not better than 1 to 5%. For this level of accuracy to be reached in the inversion 10 to 20 iterations are usually sufficient.

1 The relation between phases and the slope is given approximately by $\frac{d \log \rho_{ap}(\omega)}{d \log \omega} = \frac{4}{\pi} \phi - 1$ where ϕ is phase (Bostick 1977)

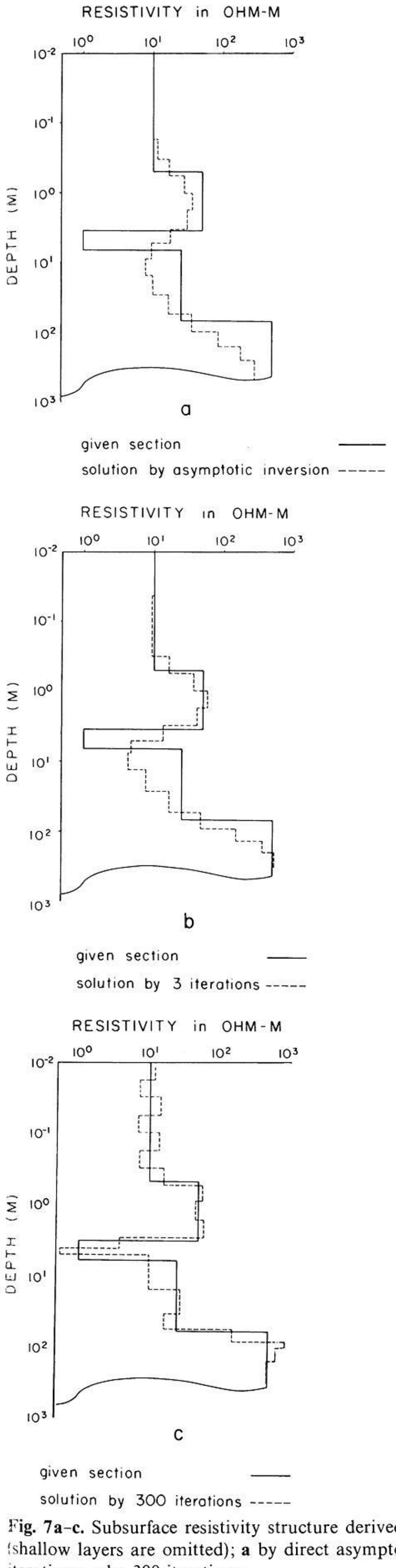

Fig. 7a–c. Subsurface resistivity structure derived for synthetic model (shallow layers are omitted); **a** by direct asymptotic inversion; **b** by 3 iterations; **c** by 300 iterations

Table 2. Original resistivity model and two possible solutions: a resistivity depth section A derived from the computed DC kernel function values and section B improved solution based on "known" resistivity value for the third layer

Layer No.		1	2	3	4	5
Synthetic	ρ (ohm-m)	10.00	50.00	1.00	20.00	500.00
model	h (m)	0.50	3.00	3.00	60.00	–
Derived	ρ (ohm-m)	9.87	57.49	0.46	27.95	493.75
section A	h (m)	0.51	2.62	1.62	69.96	–
Derived	ρ (ohm-m)	9.87	57.49	1.00	27.95	493.75
section B	h (m)	6.51	2.62	3.50	69.88	–

The corresponding resistivity-depth section reached by an asymptotic inversion is presented in Fig. 7a. The resulting sections reached after 3 and 300 iteractions are presented in Fig. 7b and c, respectively. Unlike magnetotellurics, except for shallow depth, there is a gradual improvement in the resulting solution as the number of iterations increases.

Reduction of Number of Layers – a Practical Approach to the Interpretation of DC Data

The final product of the proposed iterative process usually cannot be accepted as a final geological section because of the larger number of layers involved. Therefore, a geological interpretation based on several major geological units is still needed.

The technique of layer number reduction, presented for magnetotellurics, may also be successfully used in DC interpretation. The corresponding expressions resulting form Eqs. (4) are as follows:

$$\rho_k \Delta h_k + \rho_l \Delta h_l = \sum_{i=k+1}^{l-1} \rho_i h_i$$
$$\frac{\Delta h_k}{\rho_k} + \frac{\Delta h_l}{\rho_l} = \sum_{i=k+1}^{l-1} \frac{h_i}{\rho_i}. \tag{9}$$

The layer number reduction is illustrated in Table 2. First, a 20-layered section was derived by the iterative inversion process (300 iterations, see Fig. 7c). The accuracy reached in the fitting of kernel function curves was better than 0.5% (compared with 8% without iterations). The layer number reduction made by the proposed method only slightly effected the kernel function values, and for a 5-layered Sect. (A), the accuracy was better than 1.3%.

Section B presents a possible choice for improving the final solution using available data. It can be seen that even when a good accuracy was reached, a serious, almost 50% disagreement exists between the third layer of the given section and the derived one (Sect. B). In order to improve the final solution, a 1 ohm-m, zero thickness layer was added at an appropriate depth to the original 20-layered section. Then the number of layers was reduced using the usual method. The final solution (B) was consequently improved and is closer to the given one than section A. The accuracy in the fitting of the kernel function curves is still better than 1.3%, the same as for Sect. A.

The possibility of achieving a good fit to a given set of data with a number of distinctly different resistivity sections, illustrates a well known limitation of the DC sounding method, i.e. the existence of a wide range of equivalent solutions.

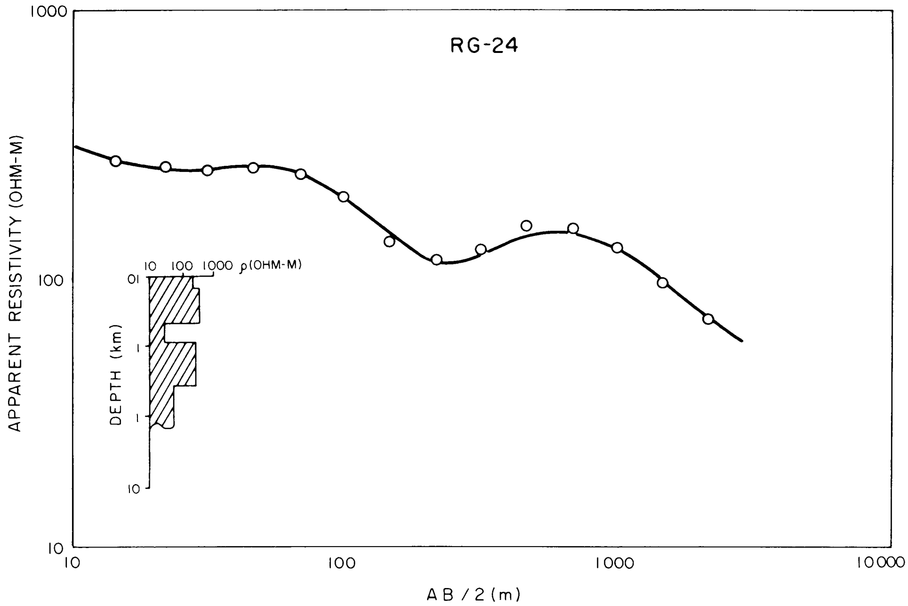

Fig. 8. Field example for DC apparent resistivity curve fitting

An advantage of the proposed method is the possibility of easily creating a wide range of equivalent solutions and choosing the most suitable one. The range of acceptable solutions may be narrowed down by using the available geological information, as illustrated in the discussion on synthetic sounding.

The scheme of inversion and interpretation of DC data is similar to the one presented for MT. Since the inversion is made on kernel function values, an appropriate transformation of the field apparent resistivity data must be done first (Koefoed 1968; Gosh 1971).

A field example is presented in Fig. 8. The apparent resistivity curve drawn corresponds to the derived resistivity section and it illustrates the good fit reached by the proposed method.

Summary

A simple and yet very effective technique for inversion of MT and DC data, using approximate equations together with exact forward computation for checking the solutions was described. This technique was extensively used in interpretation of field MT and DC data. It was found to be very effective and inexpensive.

The algorithm developed may be programmed easily using a small amount of computer storage; the computer time needed for inversion on a large computer is negligible in all practical cases. When applied on a CDC 6600 computer, inversion of usual MT and DC curves, i.e. 10 iterations of up to 15 samples lasts about half a second. An additional few milliseconds are required for layer number reduction and final interpretation.

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