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The Theory of Particle Acceleration in Astrophysical Objects Containing Shock Waves and Turbulent Plasma Motions*

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Abstract. The theory of fast charged particle acceleration by an ensemble of large-scale plasma motions with randomly distributed shock waves (i.e. supersonic turbulence) is developed. We have considered the effect of suprathermal particle acceleration in a compressible plasma with "frozen in" magnetic field and turbulent motions of different scales. The kinetic equation in the diffusion approach and diffusion coefficients in momentum space which describe the fast particle acceleration effect are obtained. There is a second order effect of Fermi acceleration in statistically uniform (non-expanding) systems with supersonic turbulence, although a single shock front leads to first order Fermi acceleration. This result is connected with the cancelation of the first order acceleration effect of shock fronts by particle deceleration in rarefaction waves between shocks.

The gain times for this acceleration mechanism are estimated. In some cases particle acceleration by supersonic turbulence is able to exceed the adiabatic deceleration losses. Some observational evidence and theoretical speculations relevant to the existence of supersonic turbulence in extragalactic radio sources, the interstellar medium, supernovae remnants and the solar wind are discussed briefly. We consider also some plausible applications of this acceleration mechanism.

Key words: Particle acceleration – Shock waves – Astrophysical supersonic turbulence

Introduction

There are several interesting works devoted to the problem of fast particle acceleration by astrophysical shock waves (Axford et al. 1977; Krymskiy 1977; Bell 1978; Blandford and Ostriker 1978). It has been shows that a strong plane shock wave propagating through turbulent plasma accelerates suprathermal particles and a power-law spectrum for the fast particles with a universal exponent is created for a wide range of initial parameters. The multiple interaction of fast particles with the shock front is provided by particle scattering on both turbulent plasma motions and "self-excited" magnetohydrodynamic waves. On the other hand there are many astrophysical objects where the existence of large scale turbulent plasma motions with shock waves seems highly likely. As we shall show below, the existence of supersonic turbulence is plausible for the jet models of extended radio sources, shock waves, overlapping regions in interstellar medium, young and old supernova remnants and for stars with strong stellar winds. Our goal is now to study the acceleration of fast particles by supersonic turbulence with shock waves.

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The Acceleration Mechanism

Let us consider fast charged particle interaction with an ensemble of randomly distributed shock waves and turbulent large-scale plasma motions with a "frozen-in" magnetic field. We use the following scale separation: large-scale plasma motions have scale $L \gg \Lambda$, where Λ is the fast particle scattering length. In our case scale L has an order of magnitude equal to the mean distance between shock fronts. We shall assume the presence of the regular magnetic field $\mathbf{B}_0$ and small-scale subsonic turbulence which provides fast particle scattering between shock fronts. The acceleration of fast particles by small scale subsonic turbulence has been studied in detail (see Tverskoy 1967; Toptyghin 1973), and we do not discuss it in this article. In this section we also suggest the following relations between the space scale sizes: $R_0 \gg L \gg \Lambda \gg R_L \gg \delta$ where R_0 is the system scale size, R_L are the gyroradii of fast particles in the mean magnetic field $\mathbf{B}_0$ and δ is the characteristic width of the shock front. Under these assumptions the kinetics of fast particles can be described in terms of a distribution function in the diffusion approach $N(\mathbf{r}, p, t)$. The $N(\mathbf{r}, p, t)$ evolution between shock fronts is described by the transport equation in a strong magnetic field (see Bykov and Toptyghin, 1980)

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial r_\alpha} \chi_{\alpha\beta} \frac{\partial N}{\partial r_\beta} - U_\alpha \frac{\partial N}{\partial r_\alpha} + \frac{p}{3} \frac{\partial N}{\partial p} \frac{\partial U_\alpha}{\partial r_\alpha} \tag{1}$$

where $\chi_{\alpha\beta}$ is the space diffusion tensor, $\mathbf{U}(\mathbf{r}, t)$ is the large-scale plasma motion velocity field. It should be noted that, in the case of weak magnetic fields ($R_L \gg l_c$, where l_c in a small scale turbulence correlation length) the transport equation has same form (1) (Dolginov and Toptyghin 1966) but there are differences in expressions for fast particle currents. Since the transport equation (1) on a shock wave surface a priori is invalid, it is necessary to include the boundary conditions for the fast particle distribution functions into the expression. These boundary conditions for the case of the plane oblique shock wave have been obtained by means of direct micro-

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scopic calculation by Vasiljev et al. (1978) and Bykov and Toptyghin (1980). This calculation takes into account the main processes on a shock wave surface: fast particle reflection and passage and the shock front's electric field (see for more detailed discussion the review by Toptyghin, 1980). The boundary condition calculation mentioned above is valid only under the assumption $\frac{U}{Y} \ll \alpha \ll 1$ where α is an angle between magnetic field and shock waves surface S and these conditions have the form:

$$N_1 = N_2 \tag{2}$$

$$n_\alpha \left(\chi_{\alpha\beta}^{(2)} \frac{\partial N_2}{\partial r_\beta} - \chi_{\alpha\beta}^{(1)} \frac{\partial N_1}{\partial r_\beta} \right) = \frac{n_\alpha (U_\alpha^{(1)} - U_\alpha^{(2)})}{3} p \frac{\partial N}{\partial p} \tag{3}$$

on S, where indices 1 and 2 are connected with upstream and downstream regions respectively, $\mathbf{n}$ is the shock surface normal vector. It should be noted that the boundary conditions, Eqs. (2), (3) may be generalized for all α. In fact, these conditions have the same form for $\alpha = \frac{\pi}{2}$, in this case the magnetic field jump and electric field are absent. Thus for $\alpha = \frac{\pi}{2}$, $N_1 = N_2$ and condition (3) may be obtained from the fast particle current continuity condition. In the general case the boundary condition (3) may be obtained from the fast particle continuity law in the form of the transport equation (1), by integrating Eq. (1) over a small interval in the vicinity of the shock surface around a direction normal to it, under the assumption that Eq. 2 is valid. But the validity of (2) is proved for the limiting cases $\alpha \ll 1$ and $\alpha \to \frac{\pi}{2}$. Fast particle acceleration is described by the right-hand term of Eq. (3), the magnetic field geometry (angles $\alpha_{1,2}$) being included in the space diffusion coefficients $\chi^{(1,2)}$. We note also that in the case $\alpha = \frac{\pi}{2}$, the acceleration effect takes place due to a scattering center velocity jump on the shock surface (i.e. first order Fermi-type acceleration). In the case $\alpha \ll 1$ the shock front electric field acceleration (with account taken of multiple particle-front interaction) dominates. For arbitrary α both Fermi-type acceleration and fast particle acceleration by the shock electric field take place but their contribution is described by Eq. (3). In order to include the conditions (2) and (3) into the transport equation (1) we must consider the fast particle distribution function N as a continuous function on the shock surface, but the diffusion tensor $\chi_{\alpha\beta}$, velocity field $\mathbf{U}(\mathbf{r}, t)$ and fast particle diffusion current $\chi_{\alpha\beta} \frac{\partial N}{\partial r_\beta}$ as a discontinous functions with finite jumps on the shock surfaces. (Hence their space argument derivatives are singular functions.) Now, we can consider fast particle acceleration by supersonic turbulence in terms of the transport equation. The distribution function N oscillations correlates with the random velocity field $\mathbf{U}(\mathbf{r}, t)$ and we must average N over the large scale motions. The method of averaging depends on the value of a dimensionless parameter $\beta = \frac{UL}{\Lambda Y} = \frac{\tau_d}{\tau_c}$, where τ_d is the characteristic diffusion time $\tau_d = \frac{L^2}{\chi}$, τ_c is the characteristic convective time scale $\tau_c = \frac{L}{U}$. If the fast particle scattering length Λ is high enough that $\beta \ll 1$, then we can use the quasilinear approach (Vedenov et al. 1962) for the averaging of the transport equation (1).

Let us separate out the slowly changing and the fast oscillating parts of the distribution function N·

$$N = F + \tilde{N}, \quad F = \langle N \rangle, \ \langle \tilde{N} \rangle = 0, \ \tilde{N} \ll F \tag{4}$$

where $\langle \ \rangle$ denotes averaging over the ensemble of the large scale turbulent motions. After substituting expression (4) for N into Eq. (1) and averaging it over the ensemble we obtain

$$\frac{\partial F}{\partial t} = \frac{\partial}{\partial r_\alpha} \langle \chi_{\alpha\beta} \rangle \frac{\partial F}{\partial r_\beta} - U_{0\alpha} \frac{\partial F}{\partial r_\alpha} + \frac{p}{3} \frac{\partial F}{\partial p} \frac{\partial U_{0\alpha}}{\partial r_\alpha} + \frac{1}{p^2} \frac{\partial}{\partial p} p^2 D \frac{\partial F}{\partial p} \tag{5}$$

where $\langle \chi_{\alpha\beta} \rangle$ is the average space diffusion tensor, $\mathbf{U}_0$ is the average velocity field and $\frac{\partial U_{0\alpha}}{\partial r_\alpha} = \left\langle \frac{\partial U_\alpha}{\partial r_\alpha} \right\rangle$ is the average divergence of the velocity field. It should be noted that in a statistically uniform nonexpanding system $\mathbf{U}_0$ and $\frac{\partial U_{0\alpha}}{\partial r_\alpha}$ should vanish. In other cases the corresponding terms in Eq. (5) describe the adiabatic deceleration (for expanding systems such as jets, supernova shells, stellar wind) or the first order particle acceleration (e.g. for accreting systems) and the convective transport. The effect of particle acceleration by supersonic turbulence is described by the fast particle momentum-space diffusion coefficient D:

$$D(p) = \frac{p^2}{9} \int \left\langle \frac{\partial U_\alpha}{\partial r_\alpha}(\mathbf{r}, t) \frac{\partial U_\beta}{\partial r_\beta}(\mathbf{r}', t') \right\rangle G(\mathbf{r}, \mathbf{r}', t, t') \, d\mathbf{r}' \, dt \tag{6}$$

Where $G(\mathbf{r}, \mathbf{r}', t, t')$ is the Green's function of the space diffusion equation.

For the isotropic turbulent velocity field we have the correlation function in the form of a Fourier transform:

$$\langle U_\alpha(\mathbf{r}, t) \, U_\beta(\mathbf{r}', t') \rangle = \int \left[A(\kappa, \tau) \, \delta_{\alpha\beta} + B(\kappa, \tau) \frac{\kappa_\alpha \kappa_\beta}{\kappa^2} \right] e^{i\boldsymbol{\kappa}\mathbf{x}} d\boldsymbol{\kappa} \tag{7}$$

where $\tau = t - t'$, $\mathbf{x} = \mathbf{r} - \mathbf{r}'$, $A(\kappa, \tau)$, $B(\kappa, \tau)$ are coefficients which depend on the spectrum of the turbulence.

We then obtain for the momentum-space diffusion coefficient:

$$D \approx \frac{p^2}{9\chi} \int [A(\kappa, 0) + B(\kappa, 0)] \, d\boldsymbol{\kappa}$$

or

$$D \approx \frac{U_{\text{eff}}^2 p^2}{9\chi} \tag{8}$$

U_{eff}^2 has an order of magnitude equal to $\langle U^2 \rangle$ and $\chi \approx \frac{Y\Lambda}{3}$.

Now we consider the case when $\beta \gg 1$, which is realized for small enough fast particle scattering length Λ. Under this condition the convective term in Eq. (1) dominates and the diffusion term can be neglected. Thus Eq. (1) reduces to

$$\frac{\partial N}{\partial t} = -U_\alpha \frac{\partial N}{\partial r_\alpha} + \frac{p}{3} \frac{\partial N}{\partial p} \frac{\partial U_\alpha}{\partial r_\alpha}. \tag{9}$$

Equation (9) can be averaged directly by means of the method which is used in invariant perturbation theory in quantum electrodynamics (Abrikosov et al. 1962). We reduce the transport Eq. (9) to the integral equation:

$$N(\mathbf{r}, p, t) = -\int_0^t d\tau \left[-\frac{\partial U_\alpha}{\partial r_\alpha}(\mathbf{r}, \tau) \frac{p}{3} \frac{\partial}{\partial p} + U_\alpha(\mathbf{r}, \tau) \frac{\partial}{\partial r_\alpha} \right] N(\mathbf{r}, p\,\tau) + N(\mathbf{r}, p, 0). \tag{10}$$

The iteration solution of (10) has the form:

$$N(\mathbf{r},p,t)=\hat{\mathrm{T}}\exp\left\{-\int_0^t d\tau\left[-\frac{\partial U_\alpha}{\partial r_\alpha}\frac{p}{3}\frac{\partial}{\partial p}+U_\alpha\frac{\partial}{\partial r_\alpha}\right]\right\}N(\mathbf{r},p,0) \qquad (11)$$

where $\hat{\mathrm{T}}$ is the "chronological ordering" operator. Since $\langle N(\mathbf{r},p,0)\rangle=N(\mathbf{r},p,0)=F(\mathbf{r},p,0)$

we obtain

$$F(\mathbf{r},p,t)=\left\langle\hat{\mathrm{T}}\exp\left\{-\int_0^t d\tau\left[-\frac{\partial U_\alpha}{\partial r_\alpha}\frac{p}{3}\frac{\partial}{\partial p}+U_\alpha\frac{\partial}{\partial r_\alpha}\right]\right\}\right\rangle F(\mathbf{r},p,0).$$

For the isotropic velocity field ($\langle U_\alpha(\mathbf{r},\tau)\rangle=0$) with Gauss distribution of amplitudes we can average the operator in expression (12) using the following relations:

$$\left\langle\hat{\mathrm{T}}\prod_{i=1}^{n}\left[-\frac{\partial U_\alpha}{\partial r_\alpha}(\mathbf{r},\tau_i)\frac{p}{3}\frac{\partial}{\partial p}+U_\alpha(\mathbf{r},\tau_i)\frac{\partial}{\partial r_\alpha}\right]\right\rangle$$
$$=\sum_{\{p\}}\prod_{i,j=1}^{n}\left\langle\hat{T}\left[-\frac{\partial U_\alpha}{\partial r_\alpha}(\mathbf{r},\tau_i)\frac{p}{3}\frac{\partial}{\partial p}+U_\alpha(\mathbf{r},\tau_i)\frac{\partial}{\partial r_\alpha}\right]\right.$$
$$\left.\times\left[-\frac{\partial U_\beta}{\partial r_\beta}(\mathbf{r},\tau_j)\frac{p}{3}\frac{\partial}{\partial p}+U_\beta(\mathbf{r},\tau_j)\frac{\partial}{\partial r_\beta}\right]\right\rangle \qquad (13)$$

for $n=2\kappa$, and it drops to zero for $n=2\kappa+1$. $\sum_{\{p\}}$ is the sum over the manifold of permutations $\{p\}$ of the natural numbers $\{1,2,\ldots,n\}$

$$\left\langle U_\alpha(\mathbf{r},\tau_i)\frac{\partial U_\alpha}{\partial r_\alpha}(\mathbf{r},\tau_j)\right\rangle=\left\langle U_\alpha(\mathbf{r},\tau_i)\frac{\partial U_\beta}{\partial r_\beta}(\mathbf{r},\tau_j)\right\rangle=0.$$

Since we have no vectors in our problem after averaging ($\mathbf{B}_0$ is the pseudovector).

$$\langle U_\alpha(\mathbf{r},\tau_i)\,U_\beta(\mathbf{r},\tau_j)\rangle=\phi_{\alpha\beta}(\tau_i-\tau_j) \qquad (14)$$

$$\left\langle\frac{\partial U_\alpha}{\partial r_\alpha}(\mathbf{r},\tau_i)\frac{\partial U_\beta}{\partial r_\beta}(\mathbf{r},\tau_j)\right\rangle=-\left\langle U_\alpha(\mathbf{r},\tau_i)\frac{\partial^2 U_\beta}{\partial r_\alpha\partial r_\beta}(\mathbf{r},\tau_j)\right\rangle=\Psi(\tau_i-\tau_j). \qquad (15)$$

Carrying out the calculations and taking into account (13), (14), (15) we obtain:

$$F(\mathbf{r},p,t)=\exp\left\{\frac{1}{2}\int_0^t d\tau_i\int_0^t d\tau_j\left[\phi_{\alpha\beta}(\tau_i-\tau_j)\frac{\partial^2}{\partial r_\alpha\partial r_\beta}+\frac{1}{9}\frac{\Psi}{p^2}\frac{\partial}{\partial p}p^4\frac{\partial}{\partial p}\right]\right\}F(\mathbf{r},p,0) \qquad (16)$$

or in standard form:

$$\frac{\partial F}{\partial t}=\frac{\partial}{\partial r_\alpha}\chi_{\alpha\beta}\frac{\partial}{\partial r_\beta}F+\frac{1}{p^2}\frac{\partial}{\partial p}Dp^2\frac{\partial F}{\partial p} \qquad (17)$$

where

$$\chi_{\alpha\beta}=\int_0^t\langle U_\alpha(\mathbf{r},t)\,U_\beta(\mathbf{r},t-\tau)\rangle d\tau \qquad (18)$$

$$D(p)=\frac{p^2}{q}\int_0^t\left\langle\frac{\partial U_\alpha}{\partial r_\alpha}(\mathbf{r},t)\frac{\partial U_\beta}{\partial r_\beta}(\mathbf{r},t-\tau)\right\rangle d\tau. \qquad (19)$$

The estimates of (18) and (19) under general assumptions about supersonic turbulence statistical properties give us

$$\chi\approx\langle U^2\rangle\tau_{cor},\qquad D\approx p^2\frac{\langle U^2\rangle}{9L^2}\tau_{cor} \qquad (20)$$

where $\tau_{cor}\approx\frac{L}{U}$ is the correlation time for large scale motions.

Finally we have $\chi\approx UL$, $D\approx\frac{U}{9L}p^2$

$$D(p)=\frac{\langle U^2\rangle p^2}{9\chi}. \qquad (21)$$

We can now summarize the results for particle acceleration by large-scale supersonic turbulence in systems with strong scattering of fast particles by small scale turbulence ($\Lambda\ll L$).

1) We have the second order Fermi acceleration effect with fast particle gain time $\tau_a\approx\frac{p^2}{D}=\frac{9\chi}{\langle U^2\rangle}$, see Eqs. (8) and (21).

2) The acceleration effect is connected with compression of the ambient medium. If $\frac{\partial U_\alpha}{\partial r_\alpha}=0$, it drops to zero.

3) The momentum space diffusion coefficient $D(p)=\frac{\langle U^2\rangle p^2}{9\chi}$ is a continuous function of the dimensionless parameter $\beta=\frac{UL}{Y\Lambda}$.

4) This acceleration effect is able to exceed the adiabatic deceleration losses of fast particles for an expanding system since e.g. for $\beta\gg1$

$$\tau_a^{-1}\simeq\frac{U}{L},\qquad\tau_{ad}^{-1}\simeq\frac{U}{R_0}$$

and $\tau_a^{-1}>\tau_{ad}^{-1}$ because $L<R_0$.

Next we consider fast particle acceleration by supersonic turbulence with weak scattering of fact particles by small-scale subsonic turbulence between shocks i.e. the case $\Lambda\gg L$. Gurevich and Rumyantsev (1973) were the first to consider this model. The isotropic distribution of fast particles takes place only on a scale $\gg L$; it is produced by means of interaction of fast particles with randomly distributed weak shocks ($\Delta B\ll B$). In accordance with their reasoning we can describe fast particles by an isotropic distribution function $F(\mathbf{r},p,t)$ over the system scale R_0. The $F(\mathbf{r},p,t)$ evolution is described by the transport Eq. (5) We now estimate the space and momentum-space diffusion coefficients for the particle acceleration by statistically uniform (nonexpanding) systems with weak shocks and large scale motions under conditions $L\ll\Lambda\ll R_0$. We define the scattering time as $\tau_s=\frac{\Lambda}{Y}$; it is clear that τ_s^{-1} is proportional to $\langle\mathbf{F}_m^2\rangle$ (where $\mathbf{F}_m=\frac{l}{c}\mathbf{V}\times\Delta\mathbf{B}$ is the magnetic force) since $\langle\mathbf{F}_m\rangle=0$. On the other hand particle acceleration takes place due to the action of the stochastic electric field with electric force

$$\mathbf{F}_e=-\frac{e}{c}\mathbf{U}\times(\mathbf{B}_0+\Delta\mathbf{B})$$

and thus

$$\tau_a^{-1}\simeq\tau_s^{-1}\frac{\langle\mathbf{F}_l^2\rangle}{\langle\mathbf{F}_m^2\rangle}=\frac{Y}{\Lambda}\frac{\langle U^2\rangle}{Y^2}\frac{B_0^2}{\langle\Delta B^2\rangle}. \qquad (22)$$

Let us estimate the fast particle scattering length Λ in this case. The fast particle pitch angle θ change on the single weak shock front is $\overline{\Delta\vartheta} \simeq \frac{\Delta B}{B_0}$.

Since the sign of $\Delta\vartheta$ is stochastic, the mean change $\langle\Delta\vartheta\rangle$ will be equal to unity only after $N \simeq \frac{1}{\Delta\vartheta^2}$ interactions of the fast particle with weak shocks. If the mean distance between random distributed shock waves is L then we obtain $\Lambda \simeq NL \simeq \frac{B_0^2}{\langle\Delta B^2\rangle} L$.

Thus in accordance with (22) we can write

$$\tau_a^{-1} \simeq \frac{\langle U^2\rangle}{YL} = \frac{U_{sh}^2}{Y\Lambda} \tag{23}$$

where $U_{sh}^2 = \langle U^2\rangle \frac{B_0^2}{\langle\Delta B^2\rangle}$ is the mean square weak shock velocity. Finally we have

$$\chi \approx YL \frac{B^2}{\langle\Delta B^2\rangle} \quad \text{and} \quad D(p) \simeq \frac{U^2 p^2}{\chi} \tag{24}$$

i.e. the second order Fermi acceleration effect. Particles acceleration by an ensemble of weak shock fronts under the condition $\Lambda \gg L$ has been considered by Gurevich and Rumyantsev (1973, 1978). They have obtained the following estimate for the fast particle gain time

$$\tau_a^{-1} = \frac{U_{sh}}{4L} \frac{\Delta B}{B_0} \tag{25}$$

This estimate (25) differs from our estimate (23) since they have obtained the first order Fermi acceleration effect. We believe this difference is due to the incorrect allowance for rarefaction waves between shock fronts made by Gurevich and Rumyantsev. It should be noted that in all cases which we describe above the second order Fermi acceleration takes place for the fast particle acceleration by supersonic turbulence in statistically uniform, nonexpanding systems. This fact is connected with complete cancellation of the first order particle acceleration effect (which takes place on a single shock front in a turbulent medium) by adiabatic losses in rarefaction waves between shocks. The presence of such rarefaction waves is necessary for nonexpanding (or nonconverging) systems.

Discussion and Some Astrophysical Applications

In this paragraph we briefly discuss some problems which are connected with supersonic turbulence development and fast particle acceleration in astrophysical objects (see also Bykov and Toptyghin 1981).

The Extragalactic Radio Sources

There is an attractive theory for the explanation of extended radiogalaxy structure – the beam model (Blandford and Rees 1978). It was proposed that collimated supersonic flow from the active nuclei of galaxies provides, by means of interaction with large plasma clouds, the observed structure of extended radio sources. This model is a plausible explanation of the radio and optical structure of the jet in M 87 (see the optical Knots model by Blandford and Königl 1979) and the structure of the galactic stellar object with peculiar line emission – SS 433 (Begelman et al. 1980). In such systems the development of supersonic turbulence takes place on the interaction surfaces of supersonic jets and plasma clouds (e.g. supernova shells). The presence of jet inhomogeneities (sound and entropy waves, etc.) leads naturally to a rising of shock waves by means of breaking (Rees 1978) and interaction of these waves with the cloud's bow shock continuously generates the supersonic turbulence within the cloud's surface layer. The small-scale subsonic turbulence would be generated by means of interaction of linear magnetohydrodynamic waves with the bow shock. Thus such systems may effectively re-accelerate the fast particles associated with a strong bow shock.

Cosmic Ray Acceleration in the Interstellar Medium

One of the main problems in the theory of the origin of cosmic rays is whether most cosmic rays are accelerated in localized sources or in the interstellar medium (ISM). It seems highly likely that a fast particle acceleration mechanism by large-scale plasma motions and shock waves plays an important role for both classes of models since most of the available energy storage is in large-scale plasma motions. A detailed model of cosmic rays re-acceleration by single shock waves in the extended envelopes of supernovae remnants has been considered recently by Blandford and Ostriker (1980). Models of cosmic ray acceleration in OB stars shells and supernovae remnants in dense clouds have been discussed by Völk (1981).

We now consider cosmic ray acceleration by ISM turbulence. There is much observational evidence for the existence of large-scale turbulence in the ISM. A study of interstellar motions by Larson (1981) showed that the velocity dispersion σ in molecular clouds follows an approximate power-law dependence $\sigma = 1.1(L)^{0.38}$ km/s where L is the scale size of the observed region measured in pc. Such dependence holds for $0.1 \lesssim L \lesssim 100$ pc. It is suggested that the observed motions are all part of a common hierarchy of interstellar turbulence. Large-scale fluctuations ($L \sim 100$ pc) of the interstellar magnetic field were reviewed by Heiles (1976). It seems highly likely that large-scale ISM turbulence sources are shearing flows from differential rotation of galactic gas with spiral shock waves and nonstationary phenomena such as supernovae explosions etc. The essential point is that the ISM gas energy budget is regulated by supernova shock-wave propagation through the inhomogeneous ISM. Supernovae explosions produce an interconnecting "tunnel" system in the ISM (see review by McCray and Snow 1979). Large-scale turbulence with randomly distributed shocks in the ISM tunnels, probably with coronal gas, is developed due to strong shock wave interactions with ISM inhomogenuities. In fact the shock wave interaction with randomly distributed clouds leads to secondary shock production. Small-amplitude hydromagnetic waves (e.g. generated by cosmic ray streaming instability) are amplified by strong shock waves (McKenzie and Westphal 1970). Hydromagnetic waves with wavelengths $\lambda < 10^{17}$ cm are collisionless in the ISM coronal gas phase and thus their damping is anisotropic. However, energy transfer between the different scales due to nonlinear decay is faster then collisionless wave damping for scales

$$l \gtrsim r_i \left(\frac{L}{r_i}\right)^{1/4} \left(\frac{Y_i^2}{E}\right)^{3/8}$$

where r_i and Y_i are the thermal ion gyroradius and velocity respectively, E is energy per unit mass for the main scale L of turbulence (Foote and Kulsrud 1979). Thus nonlinear decay of the regular large scale motions leads to turbulence formation in the tunnels with coronal gas. The inertial range of such turbulence is $10^{12}\,\text{cm} \lesssim l \lesssim 3 \cdot 10^{20}\,\text{cm}$, velocity for the main scale of this turbulence $U(L) \sim 100\,\text{km/s}$. The speed of sound for hot coronal gas with $(n, T) \sim (3 \cdot 10^{-3}\,\text{cm}^{-3}, 10^6\,°\text{K})$ is $C_s \sim 100\,\text{km/s}$.

We now consider cosmic ray acceleration by large-scale turbulence in the large coherent structures of ISM-tunnels. The main features considered in the model are enumerated below:

1) Cosmic ray acceleration for both primary and secondary nuclei takes place in the ISM tunnels.

2) Primary cosmic ray nuclei are injected with energy $\varepsilon_{inj} \lesssim 10^9\,\text{ev}$ by shock waves directly from a thermal pool. Interstellar grains should be destroyed by shock waves or a betatron mechanism since heavy elements are depleted in the ISM (Eichler, 1980).

3) Secondary nuclei are produced by means of interaction of primary nuclei in the ISM molecular clouds. They are then injected into the acceleration regime within the tunnels.

Cosmic ray acceleration in the tunnel is described by Eq. (5), but we now consider steady state, statistically uniform turbulence with $\mathbf{U}_0 = 0$ and $\operatorname{div} \mathbf{U}_0 = 0$. We include a source term $S(p)$ into the transport equation:

$$\frac{\partial}{\partial r_\alpha} \langle \chi_{\alpha\beta} \rangle \frac{\partial F}{\partial r_\beta} + \frac{1}{p^2} \frac{\partial}{\partial p} p^2 D \frac{\partial F}{\partial p} = S(p). \tag{26}$$

In accordance with the consideration, detailed above fast particle diffusion and acceleration are determined by the dimentionless parameter $\beta = \frac{UL}{Y\Lambda}$. For relativistic particles there is an energy dependence of scattering length $\Lambda(\varepsilon) \sim \varepsilon^{2-\nu}$ where ν is the small-scale turbulence spectral index (Toptyghin 1973). Unfortunately the spectral index ν for ISM turbulence is unknown. But if the same value holds for hot ISM gas that is appropriate for molecular clouds i.e. $\nu = 1.76$ then $\Lambda(\varepsilon) \sim \varepsilon^{0.24}$. We can estimate $\frac{L}{\Lambda(\varepsilon)}$ from the observed anisotropy of cosmic rays since for tunnels $L \lesssim R_0$ (here R_0 is a minimum scale size of a tunnel which has an elongated form). Thus we find $\frac{L}{\Lambda(\varepsilon)} > 10^4$ for $\varepsilon \simeq 10^9\,\text{ev}$ and $\frac{L}{\Lambda(\varepsilon)} \simeq 10^3$ for $\varepsilon \lesssim 10^{16}\,\text{ev}$. Therefore we have $\beta \gtrsim 1$ for $\varepsilon < 10^{16}\,\text{ev}$ and the turbulent diffusion regime is realized with large-scale diffusion coefficient $\chi \approx UL$ and $D(p) \simeq \frac{p^2 U}{9L}$ within the tunnel's inner region. For primary nuclei we can introduce a source term of the form $S(p) = S_0(p)\,\theta\left(1 - \frac{p}{p_{inj}}\right)$ where $S_0(p)$ is an arbitrary function of momentum p which is determined by the concrete form of the injection mechanism, $p_{inj} = \frac{\varepsilon_{inj}}{c}$ with $\varepsilon_{inj} \lesssim 10^9\,\text{ev}$.

Then accelerated primary nuclei spectrum can be obtained from Eq. (26):

$$p^2 F_1(p) \sim S_0(p_{inj})\, p^{-\gamma_1}$$

$$\gamma_1 = -\frac{1}{2} + \sqrt{\frac{9}{4} + 9\left(\frac{L}{R_0}\right)^2}. \tag{27}$$

This power-law spectrum of primary nuclei extends over an energy range of at least $10^9\,\text{ev} < \varepsilon < 10^{16}\,\text{ev}$ with spectral index $\gamma_1 = 2.85$ for $L = R_0$. For $\varepsilon \gtrsim 10^{16}$, $\beta < 1$ and we have a particle acceleration - diffusion regime with $\chi \simeq \frac{Y\Lambda(p)}{3}$ and $D(p) = \frac{U^2}{9\chi(p)}$ (see Eq. (8)). The declination of the cosmic ray spectrum for $\varepsilon > 10^{16}\,\text{ev}$ would be determined by the small-scale turbulence spectral index ν since $\Lambda(p) \sim p^{2-\nu}$. Let us now consider the spectrum of secondary nuclei in this model. The matter traversed by primary nuclei during the acceleration time τ_a is determined by $\bar{\rho} c \tau_a \simeq 3 \cdot 10^{-2}\,\text{g/cm}^2$.

This value is too small for secondary nuclei generation in the tenuous tunnels gas. Therefore we assume that secondary nuclei production takes place in dense phases of the ISM e.g. in molecular clouds with $(n, T) \sim (10^3\,\text{cm}^{-3}, 10\,\text{K})$.

A key feature of this assumption is that a large fraction of ISM mass is in the molecular clouds and it is important for our model that active star formation probably takes place at the outer edges of molecular clouds. In order to determine the source term of transport Eq. (26) for the secondary nuclei we have to find the primary nuclei spectrum in the molecular cloud, since the secondary nuclei production cross-section is nearly energy independent above $\varepsilon > 10^9\,\text{ev}$. The primary nuclei spectrum within the scatter free molecular cloud may be found by means of an extension of the spectrum (27) with account taken of the boundary layer between the tunnel and the molecular cloud, it's likely scale size $\sim 10\,\text{pc}$. Fast particle leakage from the tunnel is determined by an ordinary diffusion contrary to turbulent diffusion within the inner tunnel region. Since the fast particle scattering length Λ is energy dependent, we have a declination of the primary nuclei spectrum in the molecular cloud and therefore the secondary nuclei source term has the form

$$S(p) = S_2\, p^{-(\gamma_2 + 2)} \tag{28}$$

where

$$\gamma_2 = \gamma_1 + \Delta.$$

It should be noted that, in contrast to primary nuclei spectrum formation, the secondary nuclei source term determines the spectral index of accelerated secondary nuclei over the whole range of observed energies of the secondary cosmic rays. Therefore, for accelerated secondary nuclei we have:

$$p^2 F_2(p) \sim S_2 \cdot p^{-\gamma_2}. \tag{29}$$

Thus the observed primary to secondary energy dependence ratio can be explained in our model if

$$\Delta \simeq 2(2 - \nu) \simeq 0.5 \quad \text{for } 10^9 < \varepsilon < 10^{11}\,\text{ev}.$$

Finally it should be noted that further development of our understanding of ISM turbulence structure and investigations of γ ray production in molecular clouds would make possible a more quantitative development of the model outlined here. It is also important to consider the injection problem in more detail.

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