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Short Communication

Dispersion Effect in Earth Tide Parameters?

H. Wilhelm

Geophysikalisches Institut, Universität Karlsruhe,
Hertzstr. 16, D-7500 Karlsruhe, Federal Republic of Germany

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Anelasticity of the earth's material causes attenuation and dispersion of seismic waves. The preliminary reference Earth model (PREM) introduced by Dziewonski and Anderson (1981) accounts for anelastic dispersion. With given quality factors Q_μ and Q_k the dependence of compressional and shear wave velocities α and β on the period T is expressed as follows in relation to the corresponding velocities at the reference period $T_0 = 1$ s ($T' = T/T_0$):

$$\alpha(T) = \alpha(T_0)\left(1 - \frac{\ln T'}{\pi}\{(1-E)Q_k^{-1} + EQ_\mu^{-1}\}\right) \tag{1}$$

$$\beta(T) = \beta(T_0)\left(1 - \frac{\ln T'}{\pi Q_\mu}\right) \tag{2}$$

where $E = \frac{4}{3}\frac{\beta^2}{\alpha^2}$ and T is expressed in seconds, $Q_\beta^{-1} = Q_\mu^{-1}$ and $Q_\alpha^{-1} = EQ_\mu^{-1} + (1-E)Q_k^{-1}$. Since E appears only in a correction term it can be assumed constant and equal to $\frac{4}{3}\frac{\beta^2(T_0)}{\alpha^2(T_0)}$ in the calculation of $\alpha(T)$.

Kanamori and Anderson (1977) have shown that Eqs. (1) and (2) are valid for a constant Q in the seismic frequency band. If Q is assumed to retain its constant value of the seismic frequency band also in the tidal frequency band (see e.g. Smith and Dahlen, 1981) the influence of dispersion on the tidal response of the earth can easily be determined.

With this assumption, which may not be a realistic one, Love's numbers h and k, Shida's number l, the gravimetric factor δ and the tilt factor γ have been calculated for forcing periods from 5 to 350 h for the equivalent isotropic model PREM listed in Table 2 of Dziewonski and Anderson (1981). For the calculation of these earth tide parameters the first 3 km of the model which are assumed to be composed of water are replaced by material with the properties $\alpha(T_0) = 4.66\,\mathrm{km\,s^{-1}}$, $\beta(T_0) = 2.56\,\mathrm{km\,s^{-1}}$ and density $\rho = 2.16\,\mathrm{g\,cm^{-3}}$ resulting in a slight change of gravity at the surface ($g_0 = 981.75$ gal instead of 981.56 gal). These changes are for computational ease and do not significantly affect the results.

Figure 1 shows the total dependence of the tidal parameters on the forcing period, comprising the inertia effect and the dispersion effect. In the semi-diurnal and diurnal tidal band the influence of the resonance in the free oscillations frequency band is still apparent. The liquid core resonance in the diurnal tidal band does not exist here because it is a consequence of the flattening of the core-mantle boundary and in SNREI model calculations this boundary is assumed to be spherical. In Table 1 the numerical values of the parameters are listed for different tidal periods.

The isolated velocity dispersion influence is represented in Fig. 2 by showing the difference between the parameters of Fig. 1 and the corresponding parameters calculated for PREM with frequency-independent velocities ($\alpha(T) \equiv \alpha(T_0)$ and $\beta(T) \equiv \beta(T_0)$). The differences are of the order of 10^{-3}; there is an increase of Δh, Δk, Δl and $\Delta\delta$ and a decrease of $\Delta\gamma$ with increasing period. The effects of Δh and Δk are nearly compensated in the change of the gravimetric factor $\Delta\delta$ so that in gravimeter measurements dispersion effects in the tidal frequency band remain concealed behind other disturbing influences. As Merriam (1981) has noted the optimism of Bodri and Pedersen (1980) in this respect is unfounded.

For the calculation of complex tidal Love numbers three remarks may be of interest:

1) The real parts k and μ of the complex moduli, i.e.

$$\text{compressional modulus} \quad K = k(1 + iQ_k^{-1}), \tag{3}$$

$$\text{shear modulus} \quad M = \mu(1 + iQ_\mu^{-1}), \tag{4}$$

contain frequency dependent terms of relative magnitude $O(Q_k^{-1})$ or $O(Q_\mu^{-1})$ respectively resulting from the frequency dependent parts of α and β shown in Eqs. (1) and (2).

2) The magnitude of the complex Love numbers is given by their real parts if terms of relative order $O(Q^{-2})$ are neglected. This follows from the fact that their imaginary parts are of relative order $O(Q^{-1})$ as can be shown analytically for an incompressible homogeneous earth model (compare Love 1944, pp. 257–259).

3) The real parts of the complex Love numbers can be calculated in a similar way to the Love numbers for an elastic earth model if terms of relative order $O(Q^{-2})$ are neglected. This can be shown analytically for the

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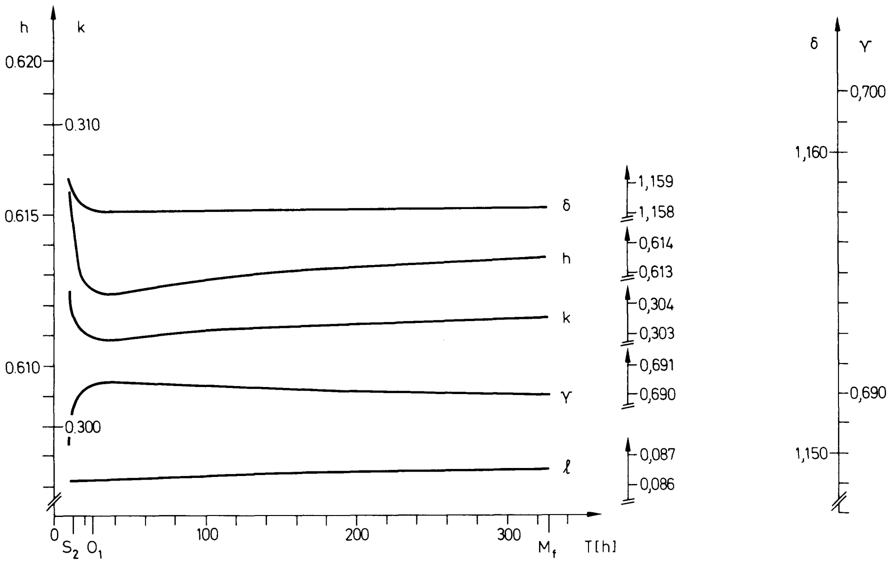

Fig. 1. Earth tide parameter dependence on period of forcing field (h and k Love's numbers, l Shida number, δ gravimetric factor, γ tilt factor) including dispersion effect for PREM (Dziewonski, Anderson, 1981)

Table 1. Earth tide parameters for different tidal constituents, calculated for PREM including dispersion effect

Constituent	Period [h]	h	k	l	δ	γ
MSm	763.487	0.61415	0.30389	0.086707	1.1583	0.68974
Mf	327.859	0.61356	0.30356	0.086549	1.1582	0.69000
O_1	25.819	0.61250	0.30290	0.086109	1.1582	0.69040
K_1	23.934	0.61256	0.30292	0.086101	1.1582	0.69036
M_2	12.421	0.61435	0.30375	0.086086	1.1587	0.68939
S_2	12.000	0.61455	0.30384	0.086090	1.1588	0.68929

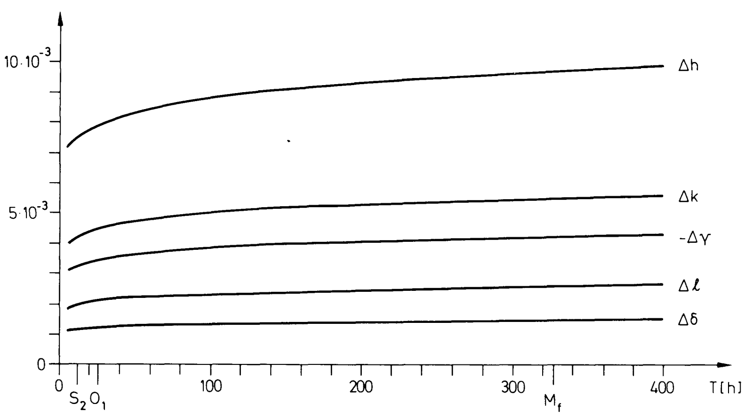

Fig. 2. Isolated dispersion effect on earth tide parameters for PREM, for explanation see text

incompressible homogeneous earth model cited above. The validity of this statement can however also be demonstrated for an inhomogeneous compressible earth model. In this case the following equations have to be solved:

$$\Delta\phi = -4\pi G(\rho\nabla\cdot\mathbf{s} + \mathbf{s}\cdot\nabla\rho) \tag{5}$$

$$\nabla\cdot T - \rho\nabla\phi + \nabla(\rho\,\mathbf{g}\cdot\mathbf{s}) - \mathbf{g}\nabla\cdot(\rho s) + \omega^2\rho s = 0 \tag{6}$$

where

$$\nabla\cdot T = (k + \tfrac{4}{3}\mu)\nabla\nabla\cdot\mathbf{s} - \mu\nabla\times\nabla\times\mathbf{s} + \nabla\mu\cdot\nabla\mathbf{s}$$

$$+\nabla(\mathbf{s}\cdot\nabla\mu) - \mathbf{s}\cdot\nabla\nabla\mu + \nabla\cdot\mathbf{s}\nabla(k - \tfrac{2}{3}\mu), \tag{7}$$

G gravitational constant, ρ density, $\mathbf{g}$ gravity, ϕ disturbing potential, ω circular frequency, $\mathbf{s}$ displacement, T stress tensor.

For a dissipating earth complex moduli K and M have to be introduced by Eqs. (3) and (4) and correspondingly the disturbing potential ϕ and the displacement $\mathbf{s}$ have to be assumed to be complex with imaginary parts of relative order $O(Q^{-1})$. Then the real parts of the resulting complex equations are still of the form given by Eqs. (5), (6) and (7), containing only the real parts of ϕ and $\mathbf{s}$ if terms of relative magnitude $O(Q^{-2})$ are neglected. Actually to each radially varying function in ϕ and $\mathbf{s}$, resulting from the separation of variables, must be attributed a proper Q value which has to be determined by the solution of the imaginary parts of the complex equations.

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