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On Systematic Errors in Phase-Velocity Analysis*

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Abstract. We investigate the systematic error that is introduced into the phase and amplitude of a dispersed signal by the application of a gaussian time window. The most significant contribution to the phase error is proportional to the slope of the group delay (vs. frequency) curve and inversely proportional to the square of the window width. This error is normally negligible in the 15–100 s long-period seismic band, but can be significant at periods longer than 240 s for fundamental-mode Rayleigh waves. To first order, no phase error is associated with a slope of the amplitude spectrum. We derive a simple nonlinear formula that predicts the bias in the phase velocity to a few parts in ten thousand; it applies both to the moving-window and to the multiple-filter method.

Key words: Surface waves – Phase-velocity analysis

The Problem

Two methods are widely used in phase velocity analysis of seismic surface waves. The moving-window method consists in isolating parts of the wavetrain from other seismic arrivals or noise with a set of time windows, and determining the Fourier phase at each frequency in the appropriate window. As stated by the convolution theorem of the inverse Fourier transform, this procedure is (for a given window) equivalent to convolving the signal spectrum with $(2\pi)^{-1}$ times the Fourier transform of the window function. The resulting Fourier phases are in general different from the original ones whose spatial dependence defines the phase velocity. Phase velocities obtained with the moving-window method are therefore biased, and while the bias is small when the window is wide compared to the signal period, substantial errors can arise at very long periods where windows no more than a few periods wide must be used to separate subsequent surface-wave arrivals.

An alternative method for phase velocity analysis is the multiple-filter method. It consists in transmitting the seismic signal through a set of zero-phase bandpass filters, and evaluating the instantaneous phase of the filtered signals at or near the group arrival time. No time windowing is applied explicitly, but it is implied in the duration of the transient response of the filters. There are some problems, both theoretical and practical, associated with this method. The instantaneous phase is in general different from the Fourier phase, and no simple relationship exists between them unless simplifying assumptions are made (such as constant amplitude and a linear group delay "curve"). It is therefore not quite obvious how the phase velocity is related to the instantaneous phase, and in fact the instantaneous phase of an unfiltered signal gives only a poor estimate of the phase velocity. Nevertheless the multiple-filter method is equivalent to the moving-window method provided that the time at which the phase is read, and the frequency to which it is associated, are correctly identified. This follows from the identity (Eq. (1) of Kodera et al., 1976):

$$s(t_0, \omega_0) = \frac{1}{2\pi} \int F(\omega)\, H(\omega - \omega_0) \exp(i\,\omega\, t_0)\, d\omega$$

$$= \exp(i\,\omega_0\, t_0) \int f(t)\, h(t_0 - t) \exp(-i\,\omega_0\, t)\, dt. \qquad (1)$$

Here $f(t)$ is the seismic signal, $F(\omega)$ its Fourier transform, $h(t)$ is a window function which we assume to be real and symmetric with respect to the origin, and $H(\omega)$ is its Fourier transform that has the same property. $s(t_0, \omega_0)$ is an estimate for the complex signal amplitude at time t_0 and frequency ω_0; its computation is known as a frequency-time analysis (FTAN), and a two-dimensional display of its modulus is known as a Gabor matrix.

The time-domain integral represents the Fourier amplitude, at frequency ω_0, of the signal $f(t)$ in a time window centered at time t_0. The frequency-domain integral is an inverse Fourier transform; it represents the instantaneous complex amplitude, at time t_0, of the signal that was passed through a zero-phase bandpass filter centered at ω_0. Equation (1) states that the two complex amplitudes are equal when the phases are referred to a common time origin. Thus, the moving-window and multiple-filter methods of frequency-time analysis are mathematically equivalent, and the phase corrections which we are going to derive apply to both methods.

Apart from a phase factor that refers the phase to t_0 as the time origin, the estimate $s(t_0, \omega_0)$ mathematically approaches the complex Fourier amplitude $F(\omega_0)$ when we increase the window width to infinity so that $H(\omega)$

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in Eq. (1) becomes 2π times a delta function. Vice versa, $s(t_0, \omega_0)$ becomes the instantaneous value $f(t_0)$ when the time window $h(t)$ is reduced to a delta function. In the analysis of real signals, the presence of noise would prevent us from reaching these limits; the integrals would diverge away from the desired values when the window width, or the bandwidth, are unreasonably increased. We therefore need methods to convert FTAN phase estimates obtained with a limited set of windows or filters into Fourier phases (for a phase velocity measurement) or into instantaneous amplitudes and phases (for a group velocity measurement).

Dziewonski et al. (1972) and Denny and Chin (1976) have proposed methods to measure group velocities of dispersed seismic signals without bias. The residual-dispersion method of Dziewonski et al. eliminates the phase bias as well, without determining it explicitly. Still, an independent investigation of the bias remains desirable, whether for the purpose of determining its magnitude, to derive criteria for the selection of windows, or for correcting phase velocities obtained with different methods. We have not found any useful formula for the difference between FTAN phase estimates and Fourier phases in the seismological literature. Such formulae were however developed in unpublished investigations by Nyman (1977) and Cara (1978) for gaussian multiple filters. Nyman takes into account third-order derivatives of the signal spectrum with respect to frequency, but assumes that the instantaneous phase is evaluated at the exact group arrival time, which is in general unknown. Cara solves the problem to second order in the phase and to arbitrary order in the amplitude spectrum with an infinite series. Their results cannot be compared to ours term by term due to their different mathematical form, except for the leading first-order term which agrees with our Eq. (13) in each case. The inclusion of derivatives higher than the second appears unrealistic from a practical point of view. We therefore restrict our investigation to derivatives of second order of the logarithmic signal spectrum. The resulting error formula (10) has a comparatively simple structure and permits us to discuss in some detail the functional dependences involved, especially the influence of the position and width of the window on the FTAN phase. The second-order formula is sufficiently accurate for practical applications in surface-wave seismology. Accuracy is however not our main point. As will be discussed later, the "systematic error" consists of one desired and one undesired component; only the latter needs be removed, but the distinction is to some degree subjective. A discussion of the "accuracy" of a method for bias correction is therefore not meaningful without reference to a specific problem.

Prediction of the Systematic Error

Consider a plane dispersed wave that is recorded at a distance Δ from the source:

$$f(t, \Delta) = \frac{1}{2\pi} \int A(\omega) \exp[i(\omega t - k\Delta)]\, d\omega$$
$$= \frac{1}{2\pi} \int F(\omega) \exp(i\omega t)\, d\omega. \tag{2}$$

Assuming that the amplitude has no zeroes, we write the Fourier spectrum $F(\omega)$ in the form

$$F(\omega) = A(\omega) \exp(-ik\Delta)$$
$$= \exp[a(\omega) - i\omega\tau_\varphi(\omega)] \tag{3}$$

where $a(\omega) = \log A(\omega)$ and $\tau_\varphi(\omega) = k\Delta/\omega = \Delta/c(\omega)$. c is the phase velocity and τ_φ the phase delay time. The signal (2) is analyzed in the time window

$$w(t) = h(t - t_0) = \exp[-(t - t_0)^2/\varepsilon^2] \tag{4}$$

of width ε. This is equivalent to convolving $F(\omega)$ with

$$\frac{1}{2\pi} W(\omega) = \frac{1}{2\pi} H(\omega) \exp(-i\omega t_0)$$
$$= \frac{\varepsilon}{\sqrt{4\pi}} \exp\left[-\frac{\varepsilon^2}{4}\omega^2 - i\omega t_0\right]. \tag{5}$$

The result at frequency ω_0 is

$$s(t_0, \omega_0) = \frac{\varepsilon}{\sqrt{4\pi}} \int \exp\left[a(\omega) - i\omega\tau_\varphi(\omega)\right.$$
$$\left. - \frac{\varepsilon^2}{4}(\omega_0 - \omega)^2 - i(\omega_0 - \omega) t_0\right] d\omega. \tag{6}$$

To solve the integral, we replace the logarithmic amplitude $a(\omega)$ and the phase $\omega\tau_\varphi(\omega)$ by their second-order Taylor expansions around $\omega = \omega_0$. This is the only approximation in the derivation of Eq. (10). Our result will therefore be exact to that degree to which $a(\omega)$ and $k(\omega)$ can be represented by second-order polynomials in the effective bandwidth of integration. Observing that $\tau(\omega) = \frac{d}{d\omega}(\omega\tau_\varphi)$ is the group delay time, we have

$$a(\omega) \cong a(\omega_0) + (\omega - \omega_0)\, a'(\omega_0) + \tfrac{1}{2}(\omega - \omega_0)^2\, a''(\omega_0), \tag{7}$$

$$\omega\tau_\varphi(\omega) \cong \omega_0 \tau_\varphi(\omega_0)$$
$$+ (\omega - \omega_0)\, \tau(\omega_0) + \tfrac{1}{2}(\omega - \omega_0)^2\, \tau'(\omega_0). \tag{8}$$

The constant term, $\exp[a(\omega_0) + i\omega_0\tau_\varphi(\omega_0)]$ can now be extracted from the integral; it represents the original Fourier coefficient $F(\omega_0)$. The remaining integral represents the error that was introduced by time-windowing.

$$\frac{s(t_0, \omega_0)}{F(\omega_0)} = \frac{\varepsilon}{\sqrt{4\pi}} \int \exp\left[(\omega - \omega_0)(a' + i(t_0 - \tau))\right.$$
$$\left. + \tfrac{1}{2}(\omega - \omega_0)^2 \left(a'' - \frac{\varepsilon^2}{2} - i\tau'\right)\right] d\omega. \tag{9}$$

From now on a, τ and their derivatives are to be taken at $\omega = \omega_0$. We will assume that $a(\omega)$ is real; the generalization to complex amplitudes (i.e., the inclusion of an initial phase at $\Delta = 0$) affects the definition of the group arrival time but does not change our subsequent derivations. $t_0 - \tau$ is the offset of the center of the window against the group arrival time τ. The integral can be evaluated with standard methods provided that a'', if positive, is less than $\varepsilon^2/2$. The result is

$$\frac{s(t_0, \omega_0)}{F(\omega_0)} = \left(1 - 2\frac{a''}{\varepsilon^2} + 2i\frac{\tau'}{\varepsilon^2}\right)^{-1/2} \exp\frac{(a' + i(t_0 - \tau))^2}{\varepsilon^2 - 2a'' + 2i\tau'}. \tag{10}$$

The error depends only on the four dimensionless quantities

$$\alpha=\frac{a'}{\varepsilon}, \quad \beta=\frac{t_0-\tau}{\varepsilon}, \quad \gamma=\frac{2a''}{\varepsilon^2}, \quad \delta=\frac{2\tau'}{\varepsilon^2}. \tag{11}$$

These normally have, in seismological applications, absolute values smaller than one, but are not always small enough to allow a linear expansion of Eq. (10). For a qualitative discussion, let us first assume that α and γ are negligible, i.e. the amplitude spectrum is flat in the vicinity of ω_0. The remaining two variables, β and δ, measure how well the window is centered at the group arrival time, and how strongly the signal is dispersed in the window. For small dispersion, $\delta \cong 0$, the right-hand side of Eq. (10) reduces to the original window function

$$\frac{s(t_0, \omega_0)}{F(\omega_0)} \cong \exp[-(\tau-t_0)^2/\varepsilon^2] \tag{12}$$

as expected for an impulsive input signal. When δ is not negligible but the window is centered, we have

$$\frac{s(t_0, \omega_0)}{F(\omega_0)}=(1+2i\,\tau'/\varepsilon^2)^{-1/2}. \tag{13}$$

The behaviour of the systematic error now depends on the magnitude of δ. This quantity determines whether a change of the signal frequency can be resolved in the window or not. The resolving power of a gaussian time window of $1/e$-width ε for angular frequencies is ε^{-1} and the variance of the angular frequency of the signal in it is $\varepsilon(2|\tau'|)^{-1}$; the latter can be resolved when $2|\tau'|<\varepsilon^2$, i.e. $|\delta|<1$. When $|\delta|$ is small, the phase error is proportional to δ and the amplitude error to δ^2. $|\delta|=1$ defines the width of an "optimum window" in which the signal has the smallest bandwidth. This is easily seen from Eq. (10) if we put $a'=a''=0$: the value $\varepsilon=\sqrt{2|\tau'|}$ minimizes the real part of the exponent, and thus the modulus of $s(t_0, \omega_0)$, at any frequency ω_0 for which the window is not centered. The Fourier transform of the optimum window defines an optimum filter of bandwidth $\sqrt{2|\tau'|^{-1}}$ for which the filtered signal has the shortest duration (Inston et al., 1971; Cara, 1973). With this optimum width, we obtain a phase bias of $\mp\pi/8$, depending on the sign of τ'. When $|\delta|$ is much larger than one, the signal is practically sinusoidal in the window; the amplitude becomes proportional to the window width, and the phase error approaches the limit $\mp\pi/4$. This is the well-known relationship between the instantaneous phase and the Fourier phase in the case of linear dispersion (compare Fig. 7.4 of Aki and Richards, 1980 and our Fig. 5).

A linear expansion of Eq. (10) is possible when all four quantities in Eq. (11) are small, i.e. when the window is sufficiently wide:

$$\frac{s(t_0, \omega_0)}{F(\omega_0)} \cong 1+\varepsilon^{-2}(a''+a'^2-(t_0-\tau)^2+2i\,a'(t_0-\tau)-i\,\tau'). \tag{14}$$

This equation indicates that for ε large and $t_0=\tau$ the phase error depends only on the window width and on the slope of the group delay curve; an eventual slope in the amplitude spectrum ($a' \neq 0$) would not cause a phase error. This is in agreement with Nyman's (1977) and Cara's (1978) formulae. Considerations of symmetry suggest that this result is not restricted to gaussian windows; any centered symmetric window should provide a phase estimate independent of the amplitude spectrum to first order. However, phase errors related to a' have occasionally been observed (Dziewonski et al., 1972; Souriau-Thevenard, 1978). Since both our Eq. (10) and Nyman's formulae predict a phase error proportional to $a'^2\tau'/\varepsilon^4$ when nonlinear terms are retained, such errors are likely to occur when the window is narrow. An alternative explanation would be that the error is not directly related to the measurement of the instantaneous phase at time t_0, but comes in when its time derivative (the instantaneous frequency) is evaluated and taken for ω_0.

Equation (14) also has an application to free mode analysis. Time-variable filtering is sometimes used to separate different modes of oscillation prior to spectral analysis. It is then essential that the window width is chosen proportional to the optimum width (i.e. to the square root of the group delay time) so that the systematic error is constant. Otherwise, the error would enter into the eigenfrequencies and the amplitude decay rates. A window of increasing width can however be realized only for the first few Rayleigh arrivals.

When the window is offset from the arrival time, Eq. (14) predicts an additional phase error proportional to the window offset and to the slope of the amplitude spectrum. Amplitude equalization, as recommended by Cara and Hatzfeld (1976) for group velocity analysis, would eliminate this part of the error, but at the same time deteriorate the signal-to-noise ratio by spectral leakage from those frequencies where the noise predominates. It is probably better to leave the amplitudes unchanged, giving automatically less weight to those frequencies where the signal-to-noise ratio is inadequate.

In practice, the width of the window must often be chosen such that none of the Eqs. (12), (13) or (14) is applicable. By splitting the logarithm of the right-hand side of Eq. (10) into its real and imaginary parts, we find that the dependence of the phase error on the window offset is quadratic, and its dependence on the window width is characterized by an arctangent function. We shall however postpone the discussion until we have presented the results of a numerical test.

As indicated above, our results apply without any change to a multiple-filter analysis with the gaussian filters

$$H(\omega-\omega_0)=\frac{\varepsilon}{\sqrt{4\pi}}\exp\left[-\frac{\varepsilon^2}{4}(\omega-\omega_0)^2\right]. \tag{15}$$

We have then to interpret ω_0 as the center frequency of the filter, and t_0 as the time at which amplitude and phase are read from the filtered seismogram.

A Numerical Test

As a test seismogram we have used a synthetic wavetrain (Fig. 1) representing a Rayleigh wave that has trav-

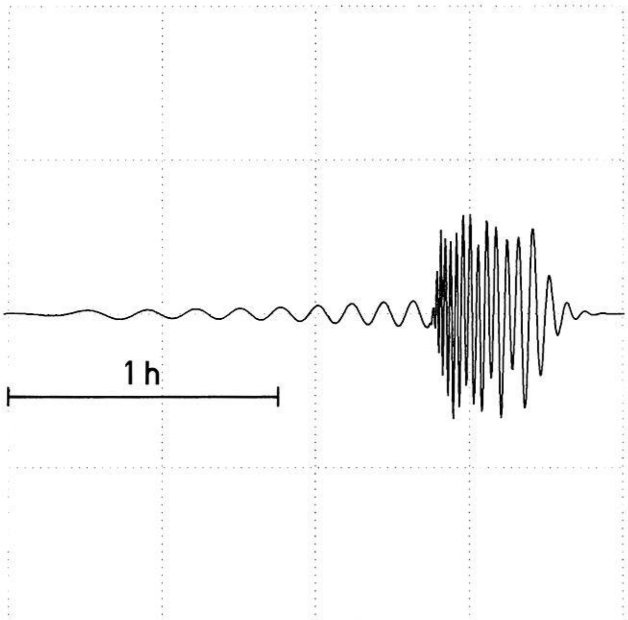

Fig. 1. The test seismogram

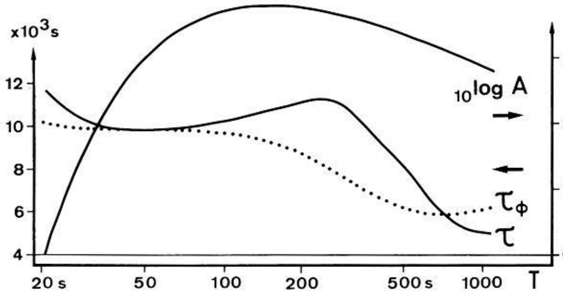

Fig. 2. Phase and group delay (lefthand scale), and log amplitude (righthand scale) of the test signal

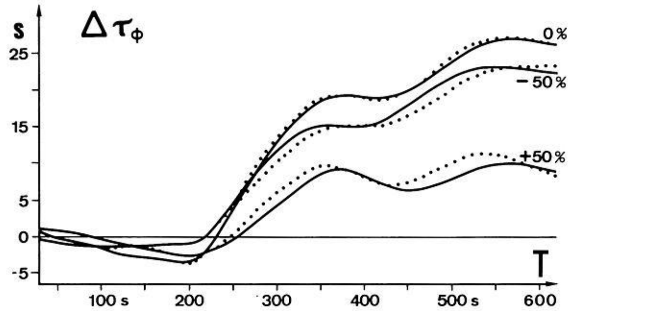

Fig. 3. Predicted and experimental errors of the phase delay time. Errors are positive when the windowed signal is delayed. Window width ε (Eq. 4) is $200\,\text{s}+2T$ at period T. The window centers are offset by -50%, 0% and $+50\%$ of the width against the theoretical group arrival time. In this figure and all following, solid lines represent experimental values and symbols predicted values

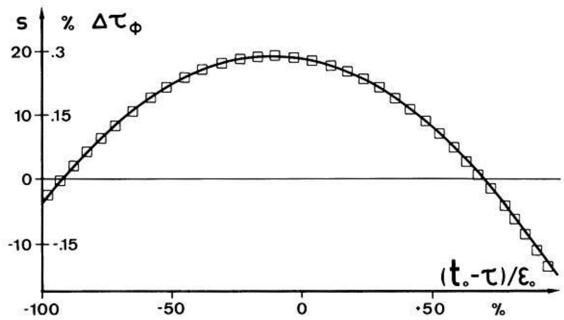

Fig. 4. Error of the phase delay at 400 s period versus window offset. 100% is an offset equal to the window width ε which is 1,000 s. The vertical scale gives the error in seconds and as a fraction of the total phase delay; with a minus sign, this is also the relative error of the phase velocity

elled over a distance of $\Delta=40{,}030$ km on a flat earth. Group velocities were fitted to those of the observed free spheroidal oscillations of the earth at periods longer than 150 s, and to those of model 1066B at shorter periods (both after Gilbert and Dziewonski, 1975). The use of cubic splines in the fit makes it possible to calculate the derivatives in Eq. (10) analytically as continuous functions. For the amplitude spectrum a simple mathematical form was specified, $A(\omega)\sim\omega^2 \exp(-\omega\Delta/2cQ)$, with a phase velocity $c=4$ km/s and a Q factor of 100 (both being relevant only at the short-period end of the spectrum). Figure 2 shows the phase and group travel times and the amplitude spectrum.

Our program for dispersion analysis evaluates Eq. (6), replacing the integral with a sum over the coefficients of the Fast Fourier Transform of the signal. This is a very efficient method that requires only one Fourier transform for any number of frequencies and windows. The sample frequencies ω_0 and the window parameters t_0 and ε at each frequency can be chosen arbitrarily. Normally we set the window width equal to $200\,\text{s}+2T$ at period T, a value that was found satisfactory in a practical application (Wielandt and Knopoff, 1982) at periods up to 400 s. At longer periods, one would probably have to use narrower windows in order to separate subsequent Rayleigh arrivals, but we have no practical experience in this.

Observed and predicted errors of the phase delay time are compared in Fig. 3 for three sets of windows: one centered at the theoretical group arrival time and one each offset by + and -50% of the width. The agreement is satisfactory, considering that the observational error per circuit is expected to be about 3 s rms for free modes (where several passes of the same wavetrain are averaged), and about twice as much for individual great-circle circuits. The systematic error is, in this example, negligible at periods shorter than 100 s, and becomes substantial only beyond 240 s. Even for window offsets as large as $\pm 50\%$, the term with $a'(t_0-\tau)$ in Eqs. (10) and (14) does not produce a significant error at the short-period end of the spectrum where the amplitude decays rapidly (Fig. 2). Experiments with less regular amplitude spectra confirm that the influence of the amplitude on the phase error is in fact very small. Since in practice amplitude and phase spectra are not independent of each other, it appears unrealistic to study their influences separately in much detail.

An unexpected result is that at long periods the systematic error is close to a maximum when the window is centered, and decreases when the window is

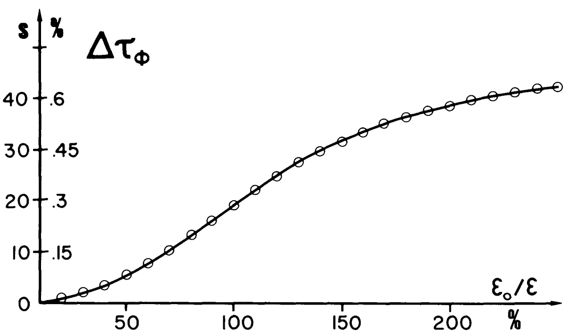

Fig. 5. Error of the phase delay at 400 s period versus window width, with no offset. Note the reciprocal scale; $\varepsilon_0 = 1{,}000$ s is the normal width

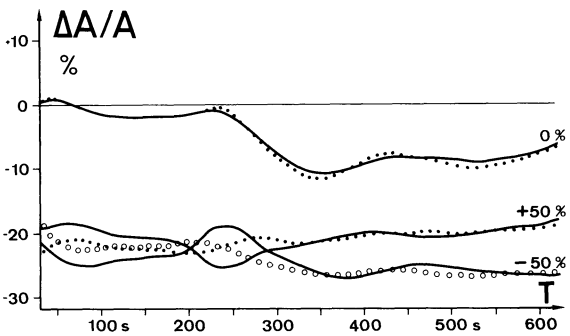

Fig. 6. Amplitude error versus period, for the same windows as in Fig. 3

offset. This is more clearly seen from Fig. 4 where the error at 400 s period is plotted versus the relative offset, $(t_0 - \tau)/\varepsilon$. For offsets of approximately -90% and $+70\%$, the systematic error vanishes in this example. We do not recommend this as a method to eliminate the bias. One might however consider adjusting the window so that the maximum of the error curve at about -10% offset, rather than the center of the window, coincides with the expected group arrival time; a small error in the latter would then not cause an uncontrolled bias.

The error in phase delay time at 400 s period as a function of the window width is plotted in Fig. 5. In order to be able to display the limiting behaviour for wide windows, we have chosen a reciprocal scale for the width, i.e. the scale is linear with respect to the bandwidth. The error increases in proportion to ε^{-2} when ε is large, but approaches a limit when the width is reduced below the normal value, as predicted by Eq. (13). The normal window width lies just between two regions of asymptotic behaviour, a fact that makes a general discussion of Eq. (10) difficult.

Figure 6 shows the relative amplitude error as a function of period for the same three sets of windows as in Fig. 3. As expected, the Fourier amplitude of the windowed signal is normally smaller than that of the original one, and reduces further when the windows are offset. Discrepancies between experimental and predicted values are somewhat larger than for the phase velocity; they are apparently related to the neglected second derivative of the group delay time.

Application to Real Signals

In contrast to the synthetic signal used to test Eq. (10), real signals often exhibit rapid oscillations in the amplitude and phase spectra for which the expansions (7) and (8) may not be accurate. One might try to include higher derivatives in the Taylor expansions; however one would then have the problem of extracting the values of these derivatives from the experimental data. Even a'' and τ' in Eq. (10) may be difficult to evaluate when the signal is noisy. For a practical application of Eq. (10), one has normally to use synthetic or smoothed experimental spectra where higher- order terms in the Taylor expansions are not very significant.

Fortunately, it turns out that the bias correction derived from a smooth spectrum is all we need. Time-windowing a dispersed signal has at the same time a desired and an undesired effect. The desired effect is the elimination of signal components and noise outside the time window; the undesired effect is the phase bias that results from spectral smoothing when the phase spectrum does not have a constant slope (i.e. the group delay is not constant). The two effects are in principle undistinguishable from each other. Only by referring to our a priori knowledge of what a dispersed seismic signal should look like are we able to define which degree of spectral smoothing is appropriate. Removing completely the phase distortion introduced by the frequency-time analysis would defeat the very purpose of this method. In general, the application of a second-order correction formula is probably a good compromise. For a more careful investigation, a scheme incorporating a differential analysis between observed and synthetic wavetrains would permit the a priori definition of the desired smoothness of the spectrum, and then the complete elimination of the systematic error for that spectrum. Remaining differences between the observed and synthetic signals would be considered as random noise whose suppression does not constitute a systematic error. The residual-dispersion method of Dziewonski et al. (1972) can be used in this way, but other methods are conceivable. It does not make much difference whether we first subtract the synthetic phase from the observed one and then analyze the resulting "residual-phase" seismogram, or first analyze the two signals separately and then form the phase difference. The effects of spectral leakage may be somewhat different, but numerical tests indicate that there is no significant difference between the two schemes with respect to their sensitivity to noise.

The systematic error can be studied directly with experimental data when a program for dispersion analysis is available that puts out a complete filtered version of the experimental seismogram. Using the latter as an input signal in the next step, we may pass the same signal through the filter repeatedly, and observe the small changes in amplitude and phase it undergoes every time. This is a very instructive experiment because it provides at the same time information on the magnitude of the systematic error and on the quality of the data (which can be judged from the stability of the error). Moreover, extrapolating backward from the phase after $n = 1, 2, 3\ldots$ passes to $n = 0$, we obtain an unbiased estimate for the unfiltered signal. The experiment (with seismograms from Wielandt and Knopoff, 1982) confirms what we have discussed above: a synthetic signal provides a valid estimate of the experimental phase bias at frequencies where the signal is good; the scatter in the phase is undesirably enhanced by removing the experimental bias at frequencies where the signal-to-noise ratio is bad.

Other Systematic Effects

While phase delays measured with plane surface waves on a stratified halfspace can be interpreted directly in terms of the velocity-depth structure, this is not so simple on a sphere. Even in the case of a laterally homogeneous, non-rotating sphere, a correction for the incomplete polar phase shift must be applied to the observed phase delay before the latter can be converted into a phase velocity (Schwab and Kausel, 1976; Wielandt, 1980). The correction depends on the source-receiver geometry and on the radiation pattern of the source, and can amount to 1% of the phase velocity at 400 s period. This correction normally has the same sign as that investigated in the present paper, so a substantial bias (in the sense of an apparent negative anomaly of the velocity) can accumulate when both are neglected. Another, although minor correction must be considered for the ellipticity and rotation of the earth (Dahlen, 1976). The purpose of all these corrections is to make phase velocities observed in different regions of the earth and with different methods comparable to each other. Their interpretation in terms of regional elastic properties of the earth is of course another, largely unsolved problem. Nevertheless, it is clear that lateral variations of the phase velocity are very small at periods between 200 and 400 s, possibly of the order of 1%. Evidence for such anomalies should not be accepted before the corrections in question have been applied, or demonstrated to be insignificant.

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