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Determination of the Steady State Temperature Gradient and of the Thermal Diffusivity in a Shallow Borehole near Braunschweig (F.R.G.)

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Abstract. In a shallow borehole of 50 m depth, drilled into a clay subsurface and located on the premises of the Biologische Bundesanstalt (Federal Institute for Biological Research) at Braunschweig, measurements of temperature as a function of depth were made from 1978–1980.

The observed temperatures were corrected for disturbances caused by known temperature variations at the surface, adopting different values of thermal diffusivities a^2. The value selected as the final one was that which gave the most linear dependence of the corrected temperature with depth. The corresponding vertical temperature gradient was used, together with a known value of heat capacity, to determine the heat flow density at the site near Braunschweig. a^2 was found to be $8.5 \cdot 10^{-3}$ cm^2/s in accordance with published values. In the same manner temperature profiles were evaluated from different depth intervals at different times. It was found that, in order to determine the geothermal temperature gradient reliably, a borehole of at least 25 m depth was required in this case. In correcting the time-dependent temperature disturbance the depth interval from 0 to 10 m should be excluded.

Key words: Geothermal temperature – Determination of thermal diffusivity – Meteorological temperature disturbances

Introduction

The determination of the geothermal temperature gradient and of the heat flow density is an important prospecting method and is described in detail in various monographs and textbooks (e.g. Kappelmeyer and Hänel, 1974; Buntebarth, 1980). In this connection reference must be made to applications in hydro-geology concerning the determination of the increased heat flow density above salt domes and also to applications in areas with increased radiogenic and chemical heat production. Knowledge of the terrestrial heat flow density is also important in the discussion of geodynamical problems.

In evaluating temperature profiles made in boreholes, three problems are generally of importance:

1. Topographic corrections must be applied to the temperature profiles (Blackwell et al., 1980).
2. For the determination of heat flow density it is preferable to use a thermal conductivity measured in situ (e.g. Musmann and Kessels, 1980).
3. Especially in the case of shallow boreholes, temperature variations penetrating from the surface into the ground must be eliminated (Lee, 1977).

To investigate this third problem in particular, temperature measurements were made from 1978–1980 in a borehole 50 m deep on the property of the Biologische Bundesanstalt at Braunschweig. In the clay encountered here, hydrological temperature disturbances can almost certainly be excluded. It was the aim of this investigation to find the minimum depth of a shallow borehole required for determining the heat flow density.

Lovering et al. (1963) and Lee (1977) corrected observed temperature profiles under the assumption of a harmonic annual temperature wave. Lee (1977) described a procedure to obtain the gradient of the temperature from a temperature profile measured at a fixed point of time.

In contrast, the temperature correction presented in this paper is in addition performed for temperature variations with periods other than annual. The correction is restricted to influences from the past 100 years. For this period the mean annual temperatures yield disturbances of the gradients of up to 30 K/km at a depth of 18 m (Kessels, 1980). Paleoclimatic corrections are not applied.

Kappelmeyer (1957) has shown that in extremely shallow boreholes (depth $\leqq 2$ m) the geothermal heat flow density cannot be measured. Near the surface, the heat flow density caused by the annual temperature wave is much greater than the heat flow density from depth. The magnitude of the heat flow density near the surface is also determined by the micro-climatic conditions in the near environment and by the geological conditions near the surface (Geiger, 1961; Kessels, 1980).

Lovering et al. (1963) demonstrated that determining the geothermal heat flow density over depths which are subject to the annual temperature wave may also have advantages over determinations in deep boreholes. This is because, in shallow boreholes, the time-dependent temperature field can be used to determine the in situ thermal diffusivity, a^2. Using a suitable assumption for the heat capacity ρc, where ρ denotes density and c denotes specific heat, the thermal conductivity λ can then be derived from:

$$a^2 = \frac{\lambda}{\rho c}$$

Lovering et al. (1963) determined the thermal diffusivity by using the point of intersection of two temperature profiles recorded at different times.

Lee (1977) determined the thermal diffusivity and the temperature gradient by a transformation of variables which transforms the non-linear temperature distribution of the annual wave in the subsurface into a linear relation. The latter is then fitted to the measured data by minimizing the standard deviation.

In the present investigation the thermal diffusivity was determined by a trial and error method, namely by calculating theoretical subsurface temperatures from measured surface temperatures together with an assumed value of the thermal diffusivity and comparing it to the temperatures observed in the borehole.

The thermal diffusivity yielding the best correspondence between the theoretical and the observed temperature profiles was taken as the final value.

In performing heat flow density measurements at the Laacher See (Eifel region, F.R.G.), Musmann and Kessels (1980) showed a good consistency between the thermal diffusivities determined in situ and those determined by fitting temperature profiles. In the measurements made at Braunschweig the surface temperatures did not have to be inferred from the air temperature since at this location the Deutscher Wetterdienst (Meteorological Service of the F.R.G.) has recorded the surface temperatures directly and continuously since 1954. The case where geothermal prospecting is to be done in an area where no direct surface temperature measurements are available was discussed in detail by Kessels (1980) and applied in the measurements at the Laacher See.

Along with the evaluation of a^2 by fitting the calculated temperatures to those observed, a proposal by Hänel (1980) for the determination of the steady state temperature gradient was considered. According to Hänel (1980) a relatively stable temperature gradient between 40 and 50 K/km may be determined from the mean surface temperature and the mean temperature at the deepest point of the borehole, both taken over two years.

Method of Analysis

The procedure presented here, which is described in more detail in Kessels (1980), assumes the subsurface to be homogeneous and isotropic, which implies that the thermal diffusivity is a constant.

The total temperature field T which depends on depth z and time t can be thought of as being split up into two parts according to

$$T(z,t) = T_s + gz + T'(z,t) \tag{1}$$

where the first two terms represent the steady-state geothermal temperature field characterized by the mean surface temperature T_s and the normal temperature gradient g, and where T' denotes the time-dependent temperature component to be eliminated by calculation. This component is caused by deviations of the surface temperature $T(0,t)$ from the long-term mean value T_s and vanishes at any depth if averaged over a long time.

For the correcting calculation usually only mean surface temperatures (mean daily, monthly, and annual surface temperatures) are available. We therefore consider the effect of a constant temperature disturbance T'_{si} imposed at the surface between times t_i and t_{i+1} in the past where $t_i < t_{i+1} < t$. According to Carlslaw and Jaeger (1959; cf. also

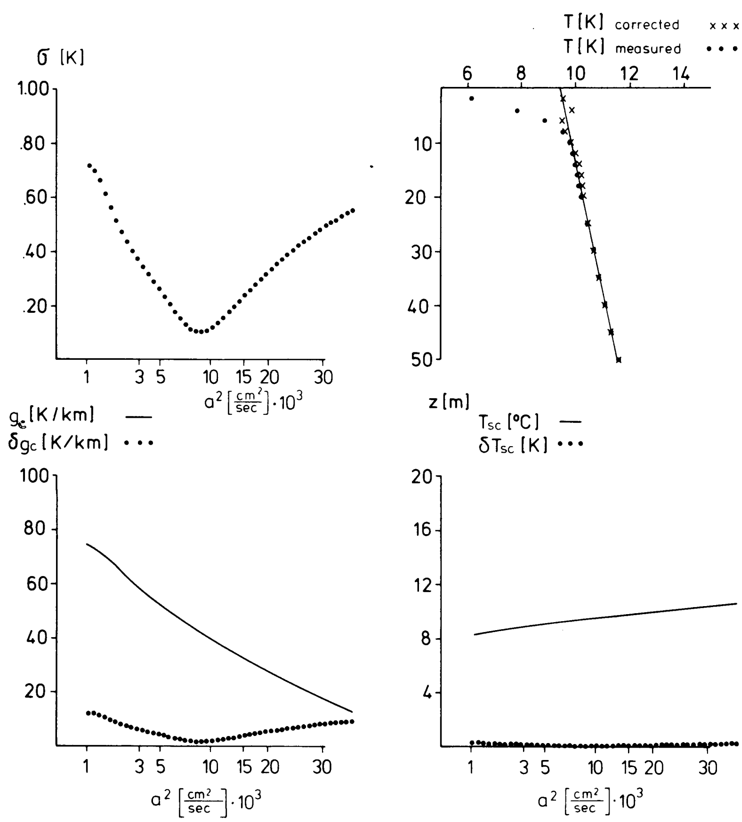

Fig. 1. Example of correction of temperatures T measured in the borehole between 2 and 50 m depth on 22 February 1980, adopting different values for the thermal diffusivity a^2. σ is the standard deviation between the corrected temperatures as depending on depth and the best fitting straight line for any value of a^2. This line is shown in the upper right diagram for $a^2 = 8.9 \cdot 10^{-3}$ cm²/s, at which value σ adopts a minimum (*upper left*). At the bottom the parameters of the best-fitting straight lines $T_c(z) = T_{sc} + g_{cz}$ are shown together with their probable errors as function of a^2. The finally adopted value of a^2 and the corresponding values of g_c and T_{sc} are indicated by the minimum of $\sigma(a^2)$

Buntebarth, 1980, p. 27) we get for the corresponding temperature disturbance at depth z and time t

$$T'(z,t)=T'_{si}\left[\Phi\left(\frac{z}{\sqrt{2}a\sqrt{t-t_{i+1}}}\right)-\Phi\left(\frac{z}{\sqrt{2}a\sqrt{t-t_i}}\right)\right] \tag{2}$$

where ϕ is the Gaussian error function

$$\Phi(x)=\sqrt{\frac{2}{\pi}}\int_0^x e^{-\frac{\xi^2}{2}}d\xi.$$

For the total time-dependent temperature disturbance we get, by summing up over all surface temperature variations in the past:

$$T'(z,t)=\sum_{i=1}^{n} T'_{si}\left[\Phi\left(\frac{2}{\sqrt{2}a\sqrt{t-t_{i+1}}}\right)-\Phi\left(\frac{2}{\sqrt{2}a\sqrt{t-t_i}}\right)\right]. \tag{3}$$

With this equation the known temperature history at the surface can be used to calculate the temperature disturbance in the subsurface for any adopted value of the thermal diffusivity a^2 and then to subtract it from the temperatures measured in the borehole. An incorrect value adopted for the thermal diffusivity will result in a nonlinear relationship between temperature and depth.

However, if the thermal diffusivity is chosen correctly, the correction will result in a linear temperature curve, around which the measured values will scatter only according to the instrumental errors of measurement.

A best-fit straight line is placed through the points of the corrected depth function (the same procedure may be applied even in the case when an incorrect thermal diffusivity is assumed), of which the standard deviation σ is then determined. σ will be minimal at the correct value of the thermal diffusivity.

In Fig. 1 the application of this procedure is demonstrated for the case of the temperature observations made in the borehole between 2 and 50 m depth. The diagram at the upper left shows that σ as a function of the adopted values of a^2 has a distinct minimum. The diagram at the upper right on the other hand shows the uncorrected temperatures as well as the optimally corrected temperatures (at minimum standard deviation). In the lower two diagrams the parameters of the best-fitting straight lines are shown as functions of a^2. These lines have the mathematical form

$$T_c(z)=T_{sc}+g_c z \tag{4}$$

which corresponds to the first two terms in Eq. (1). The subscript c indicates that these temperatures and parameters have been obtained after correction. Of course, it is hoped that T_{sc} and g_c come close to the true values T_s and g (cf. Eq. (1)) if the optimal value for a^2 has been used in the correction. The lower diagrams in Fig. 1 also give the probable errors δT_{sc} and δg_c of the parameters T_{sc} and g_c, respectively (for more details see Kessels, 1980).

Analysis of Observations 1978–1980

The procedure of analysis outlined above may be applied to data from any time t and even from any depth interval, which will be done in the following.

It should be mentioned that the measurements made from April 1978 to March 1979 were carried out with a fixed probe chain, whereas the later measurements from April 1979 to May 1980 were made with a single probe lowered into the borehole. For determining the geothermal gradient the latter method proved to be of advantage.

The example shown in the upper right part of Fig. 1 indicates that the time-dependent disturbance of the temperature field in the top few meters of the borehole cannot

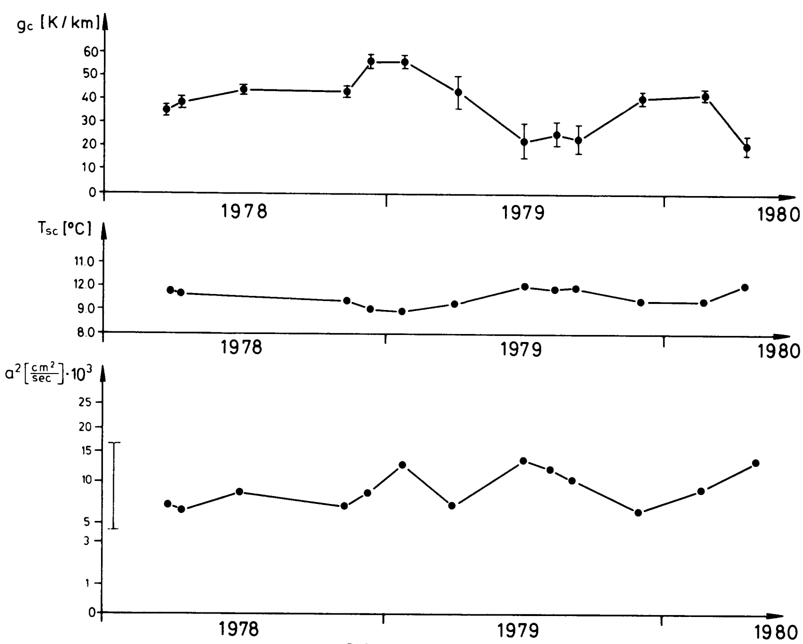

Fig. 2. Final results attained by the procedure as indicated in Figure 1 for different times. Data from the whole depth interval 2–50 m was used

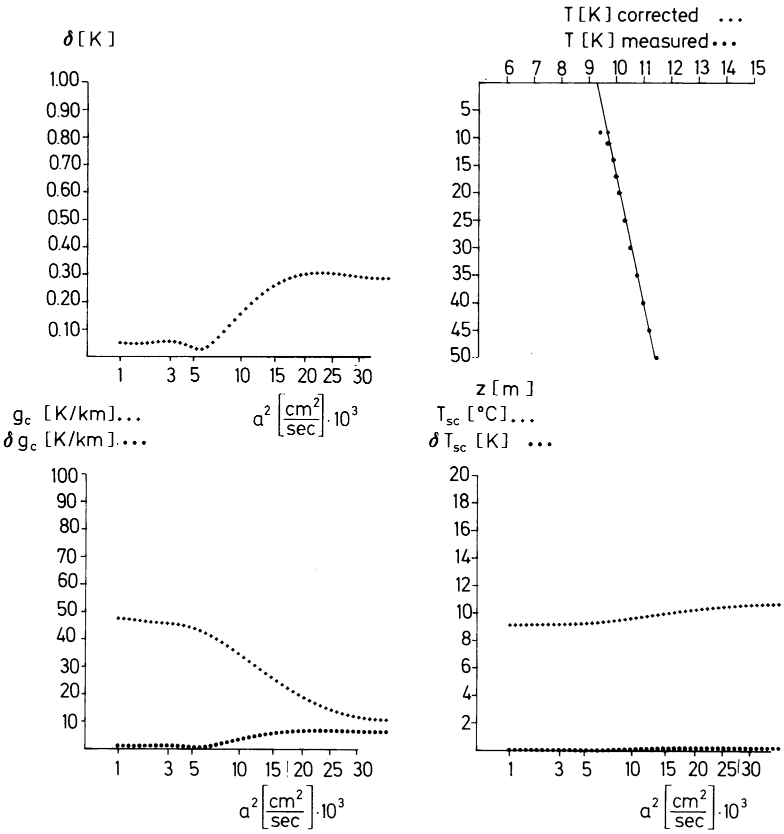

Fig. 3. Example of correction of temperatures T measured in the borehole between 10 and 50 m depth on 29 June 1979. Otherwise as Fig. 1, with the exception that $a^2 = 5.5 \cdot 10^{-3}$ cm²/s in the optimal case here

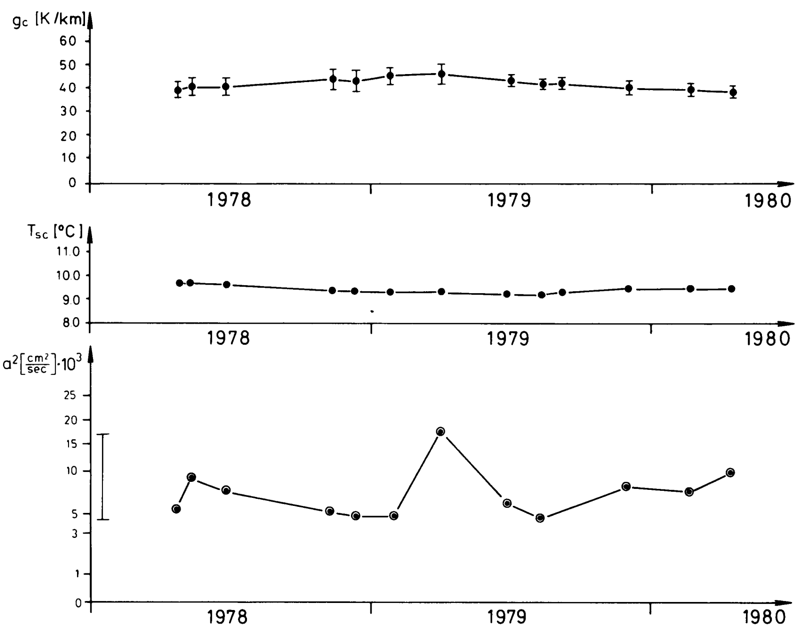

Fig. 4. Final results attained by the procedure as indicated in Figure 1 for different times. Data from the depth interval 10–50 m was used

be eliminated with sufficient accuracy, in accordance with, for example, Kappelmeyer (1957) and Lee (1977). Inhomogeneities in the subsurface, hydrological influences and the inaccuracy of meteorological data are not accounted for by the correction procedure in this depth range.

The diagram at the lower left of Fig. 1 shows that the gradient g_c determined depends heavily on the value of the thermal diffusivity used for the calculation, which implies that its final value adopted will be affected by some error.

The optimal value of the thermal diffusivity seems to be rather well defined due to the sharp minimum in the curve $\sigma(a^2)$ (Fig. 1, upper left). However, it is strongly influenced by the upper depth range, since in the upper few meters of the subsurface the temperature variations of the past few months are especially effective. This means that the thermal diffusivity determined is effectively the thermal diffusivity of the top few meters of the subsurface for a time interval of a few months prior to the measurement by a time-dependent thermal diffusivity.

Figure 2 shows the time-dependent results correspond-

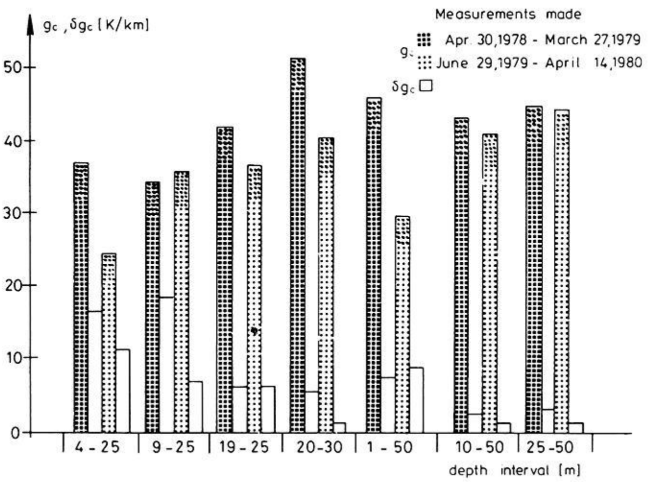

Fig. 5. Mean temperature gradients g_c as determined over different depth intervals with errors δg_c from scatter when g_c values at different times are considered

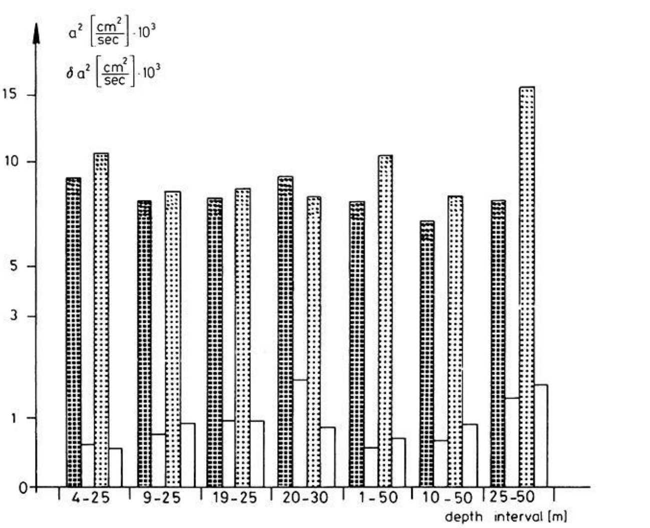

Fig. 6. Mean thermal diffusivities a^2 as determined over different depth intervals with errors δa^2 from scatter when a^2 values at different times are considered

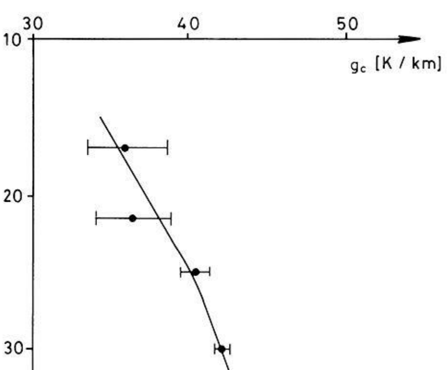

Fig. 7. Mean geothermal gradients g_c over different depth intervals at borehole site

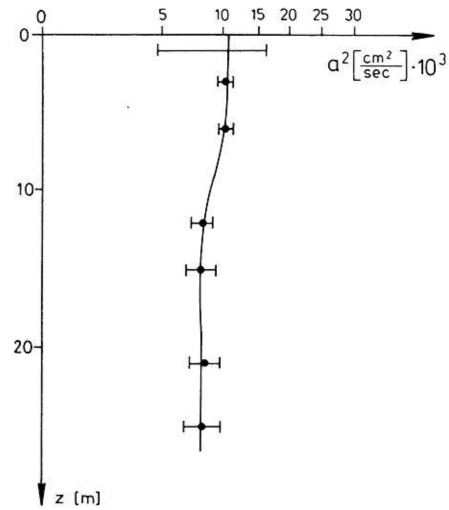

Fig. 8. Mean thermal diffusivities a^2 over different depth intervals at borehole site

ing to the values finally adopted for a^2, T_{sc} and g_c (cf. Fig. 1), where again the data from the whole depth interval 2–50 m have been used. All thermal diffusivities determined were found to be in the range known for the thermal diffusivities of clay. This range is indicated by the barred line to the left in the lower diagram (Kappelmeyer and Hänel, 1976).

The errors of the g_c values attained are greatly diminished if the upper 10 m are excluded from the analysis (Fig. 5).

For the case that the near surface temperature field (less than 10 m depth) is not included in the evaluation, the time-dependent temperature field $T'(z,t)$ (see Eq. 1) is much smaller. The result is that in the evaluation the minimum of the standard deviation, which serves to determine the thermal diffusivity, is much less distinct. A comparison of the upper left diagrams of Figs. 1 and 3 shows this clearly. The result is a smaller error of the thermal diffusivity for the depth interval 2–50 m than for the depth interval 10–50 m. This can be seen in Fig. 6. This is even more pronounced for the depth interval 25–50 m, as is also shown in Fig. 6.

For determining the temperature gradient the effect of the error of the temperature gradient is opposed to the effect of the error of the thermal diffusivity. In this case including the near-surface temperature measurements in the evaluation leads to a significant change of the corrected gradient with respect to thermal diffusivity and thus to a large error. This is evident from a comparison of Figs. 1 and 3. Comparing the different depth intervals in Fig. 5 also shows this clearly.

In Fig. 5 the results of the determination of the gradient g_c for all depth intervals are shown. As in Fig. 6, where all results for the thermal diffusivities are presented, the results are given separately for measurements with the chain probe and for the later ones with the lowerable probe. The errors indicated are not determined from the errors of the individual measurements but from the scattering around the mean values of the total time interval evaluated. These errors proved to be greater than the errors found by applying error calculations to the errors of the individual measurements. Taking into account only those depth intervals

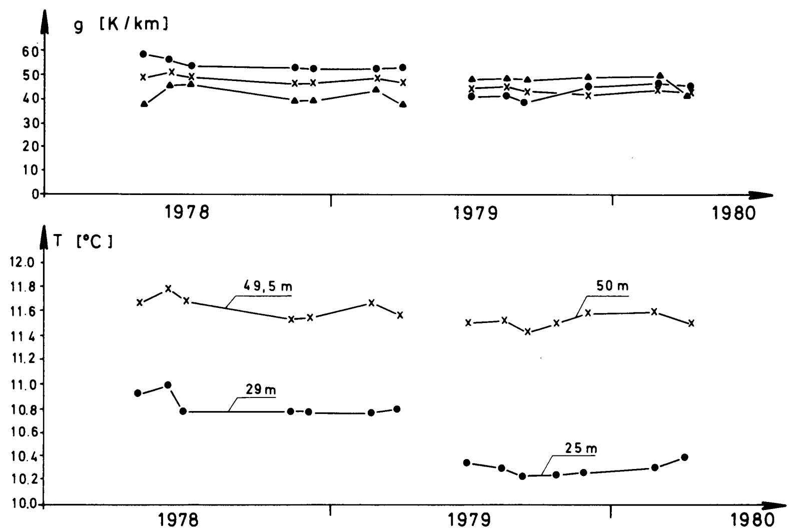

Fig. 9. Temperature gradients $g = \Delta T/z$ (*upper part*) calculated from the differences ΔT between temperatures T observed at depths z (*lower part*) and an adopted mean surface temperature of 9,2° C. *Full circles*: $z = z_1 = 29$ m or 25 m, respectively, with temperatures T_1. Crosses: $z = z_2 = 49.5$ m or 50 m, respectively, with temperatures T_2. In addition, gradients $(T_2 - T_1)/(z_2 - z_1)$ are shown, for comparison (*upper part, triangles*)

where gradients could be determined reliably, the steady state gradient can be plotted with respect to depth in the borehole. Figure 7 shows that the gradient in the top layers of the subsurface is smaller than in deeper layers. A similar profile drawn for the thermal diffusivity (Fig. 8) shows that the thermal diffusivity in the top layers is greater than in deeper layers. Altogether this indicates a tendency towards constant heat flow density as a function of depth. With the gradient and the thermal diffusivity determined by the methods outlined, the heat flow density may be calculated if the heat capacity ρc is known (Kappelmeyer and Hänel, 1974). If a value of 2.29 J/cm K is assumed for the heat capacity ρc, the heat flow density for the depth interval 1–25 m is found to be 89 ± 14 mW/m^2, whereas for the deeper interval 25–50 m it is 81 ± 15 mW/m^2. The mean heat flow density for both intervals is thus 85 ± 11 mW/m^2. This value is slightly larger than the one given by Hänel (1980) for the region of Braunschweig. This may be due to a positive heat flow anomaly connected with a salt dome (Nußberg) which is about 1 km from the borehole.

This holds particularly, since the values of Hänel are corrected for paleoclimatic effects. Such effects in our case result in an increase of the heat flow density.

The temperature data collected for this investigation was also used for testing an evaluation method proposed by Hänel (1980). Hänel proposes to determine the gradient by taking the differences between temperature values at the greatest depth of the borehole and the mean temperature at the surface. This method has the advantage that it can be performed quickly and that the temperature disturbance by drilling at the greatest depth of the borehole is already negligible a short time after the drilling is finished. Evaluation by taking differences was performed here for temperature data from the depths 50 m and 25 m, or 49.5 m and 29 m. The results are shown in Fig. 9. For the long-term mean surface temperature a value of 9.2° C was used. This value is the sum of the mean air temperature at Braunschweig given in meteorological yearbooks as 8.8° C, and the difference between surface and air temperature given by Kessels (1980) as 0.4° C.

As is shown in Fig. 9 this procedure yields relatively good results for the gradients. It must be considered that in the case of the 25 m evaluation an error of 0.1° C in the surface temperature already leads to an error of 4 K/km for the gradient. This means that the error stemming from an incorrect assumption of the long-term mean surface temperature in determining the gradient is considerable.

Concluding Considerations

The measurements made at the Biologische Bundesanstalt show that in shallow boreholes of 25 m depth or more the steady state geothermal gradient and the conductive heat flow density can be determined. In order to obtain the temperature field at greater depths by extrapolation it must be taken into account, however, that the usual model considerations must be applied. These must take into account not only the topography of the surface (Blackwell et al. 1980) but also the homogeneity and heat production of the subsurface as well as hydrological temperature disturbances. The latter are especially significant for shallow boreholes and are well suited for hydrological investigations.

In addition it should be remarked that the method proposed and applied here to correct the time-dependent temperature field is, when applied to shallow boreholes, well suited to determine the long-term mean surface temperature.

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