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ELF intensity levels at geostationary orbit and pulsating aurora

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Abstract. An expression for the strong electron diffusion condition is obtained and then used to derive the threshold ELF intensity required to put electrons in strong diffusion to explain pulsating aurora. A lower threshold than has previously been calculated (Thomas, 1983) is obtained.

A case study is then described of a night when pulsating aurorae were observed over northern Scandinavia and pulsed ELF hiss was observed on GEOS 2. It is shown that the peak field strengths of the pulsed hiss reached levels sufficient to put energetic electrons into strong pitch angle diffusion, a requirement implied by both theory and observation for explaining pulsating aurora as being caused by enhanced whistler mode wave growth rates in the equatorial plane.

Key words. Pulsed hiss – Pulsating aurora – Strong diffusion

Introduction

One consistently observed phenomenon in the post break-up phase of substorms is the occurrence of pulsations in the diffuse aurora and simultaneous Pil micropulsations (Ward et al., 1982; Ward, 1983; Heacock and Hunsucker, 1977). Of the many suggestions, e.g. Davidson (1979), Luhmann (1979), Oguti (1976), Barsukov et al. (1972), Coroniti and Kennel (1970), put forward to explain the phenomenon of pulsating aurora, the latter has received the most attention because of the circumstantial association of pulsating aurora with the spectrally similar Pil type of micropulsations. Both phenomena have similar ranges of periods (2–30 s) and occur in the post break-up phase of substorms.

The Coroniti-Kennel theory associates pulsating aurora with enhanced pitch angle diffusion of energetic electrons into the loss cone due to enhanced growth rates of ELF/VLF noise by the whistler mode instability in the presence of a micropulsation modulating the energy range, and hence the number of resonant electrons (because of the non-zero slope of the energetic electrons' distribution in velocities parallel to the geomagnetic field) available for the Doppler-shifted cyclotron resonance interaction.

They derive an expression for the precipitation rate $\dot{J}p(t)$ in the presence of a magnetic field modulation $B = B_0(1+b\sin\Omega t)$, of amplitude b and period $2\pi/\Omega$, under conditions of weak diffusion and where $2\pi/\Omega$ is several times the bounce period such that

$$\dot{J}p(t) = \dot{J}p(0)\exp\left[\frac{2bv_0}{A\Omega}(1-\cos\Omega t)\right] \tag{1}$$

where v_0 = equilibrium loss rate, A = temperature anisotropy in the energetic particles ($\simeq (T_\perp - T_\parallel)/T_\parallel$, where $T_{\perp,\parallel}$ are the mean temperatures perpendicular and parallel to the geomagnetic field direction).

Haugstad (1975) has objected to the Coroniti-Kennel formulation on the grounds that

a) Equation (1) leads to an impossibly strong dependence of $\dot{J}p(t)$ on Ω, e.g. if a 10 s sinusoidal perturbation causes a modulation in $\dot{J}p$ with a factor 2, a 100 s period perturbation implies modulation of the background precipitation rate with a factor of 2^{10} for the same value of b (Haugstad, 1975);

b) By shifting the phase of the micropulsation by π, so that $B = B_0(1-b\sin\Omega t)$, yields

$$\dot{J}p(t) = \dot{J}p(0)\exp\left[-\frac{2bv_0}{A\Omega}(1-\cos\Omega t)\right],$$

and results in the pulsations in the precipitation rate $\dot{J}p(t)$ always being negative with respect to the background. This drastic qualitative change in behaviour is difficult to accept.

c) The expression also implies a 90° phase difference between pulsations in the magnetic field and pulsations in the rate of particle precipitation.

The origin of these inconsistencies (Haugstad, 1975) lies in the assumption that no damping of the ELF waves is caused by the increased loss of resonant particles when $\dot{J}p(t) > \dot{J}p(0)$. If we define the relaxation time τ_R as the time taken for the disturbed velocity distribution to reach a new equilibrium configuration, then the assumption of no damping is equivalent to assuming $\tau_R \gg 2\pi/\Omega$. However the expressions used in deriving Eq. (1) all assume equilibrium conditions and hence that $\tau_R \ll 2\pi/\Omega$, thus implying that damping could be significant.

By including damping, hence retaining higher than first order terms in b, Haugstad derives an alternative expression for $\dot{J}p(t)$ as

$$\dot{J}p(t)=\dot{J}p(0)\exp\left(\frac{b}{A(0)}\sin\Omega t\right) \qquad (2)$$

which is derived assuming equilibrium conditions, i.e. that $\tau_R \ll 2\pi/\Omega$, and far from the weak diffusion region for Pi1 perturbation periods. He thus concludes that expressions derived from the equilibrium formulae only hold in the strong, and transition from weak to strong, diffusion regimes, and hence that the strong dependence of $\dot{J}p(t)$ upon $b\sin\Omega t$ implied by Eq. (2), which is required to account for the one to two times difference between the peak and the minimum amplitudes observed in pulsating aurora, only holds in these diffusion regimes. By including damping and thus removing the inconsistencies involved in ignoring damping while assuming that equilibrium formulae are applicable he also deduces that there is no 90° phase difference between pulsations in the magnetic field and pulsations in the rate of particle precipitation.

Rocket flights into pulsating aurorae (Whalen et al., 1971; Smith et al., 1980; Yau et al., 1981) are consistent in their observations of isotropic pitch angle distributions about the loss cone during the maxima of the pulsations, hence implying strong pitch angle diffusion. Also, satellite observations (Gough and Korth, 1982) indicate that the equatorial loss cone fills and empties in phase with increases and decreases of the intensity of ELF/VLF pulsed electromagnetic hiss, which, according to either theory, will be in phase with the magnetic field perturbations.

Hence it appears that experimental evidence supports the Haugstad approach, i.e. including wave damping due to increased losses of resonant particles. Both this theory plus the observations indicate that the wave field strength must be sufficient to put the energetic electrons into strong (or near strong) pitch angle diffusion to cause the typically observed variations in intensity of pulsating aurora.

In this paper an expression for the strong diffusion condition is derived in the manner first developed by Kennel and Petschek (1966), and then the required ELF/VLF field strength for strong diffusion is compared with observed field strengths at GEOS 2 on an occasion where pulsating auroral displays occurred over an extended period in the region near conjugate to the spacecraft.

Theory

Let $\Omega^{\pm}$ = ion/electron gyrofrequency, $\omega_p^{\pm}$ = plasma frequency, ω = wave frequency of a whistler mode wave (right-hand circularly polarized).

In the cold plasma approximation, assuming no collisions, the refractive index n is given by

$$n^2=\frac{c^2k^2}{\omega^2}=1-\frac{(\omega_p^+)^2}{\omega(\omega+\Omega^+)}-\frac{(\omega_p^-)^2}{\omega(\omega+\Omega^-)}. \qquad (3)$$

Assuming now that

i) $\Omega^+ \ll \omega < \Omega^-$,

ii) $\omega_p^- \gg \Omega^-$,

Eq. (3) reduces to

$$n^2 \simeq \frac{(\omega_p^-)^2}{\omega(\Omega^- - \omega)}. \qquad (4)$$

Such a wave can resonate with a gyrotating electron by Doppler-shifted cyclotron resonance if

$$k V_r = \omega - \Omega^-$$

where $E_r = \frac{1}{2} m_e V_r^2$ = energy of electron parallel to the magnetic field.

Hence resonance occurs when

$$E_r=\frac{B_0^2}{8\pi N_e}\cdot\frac{\Omega^-}{\omega}\left(1-\frac{\omega}{\Omega^-}\right)^3 \qquad (5)$$

where B_0 = local geomagnetic field strength and N_e = local electron density.

At the geostationary orbit and ELF/VLF frequencies (i.e. magnetic energy per particle $B_0^2/8\pi N_e \sim 25$ keV and $\omega \sim 2\times10^3$–2×10^4 rads/s) this corresponds to $E \sim 25$ to 1 keV. The higher energies ($\gtrsim 10$ keV) are the more likely to cause pulsating aurorae (precipitation altitudes $\sim$90–110 km being required). Hence the lower frequencies are the most important for providing sufficient pitch angle scattering to put these energetic particles into strong diffusion. Such a particle with pitch angle $\alpha = \tan^{-1}(v_\perp/v_\parallel)$ will have its pitch angle altered by $\Delta\alpha \simeq \Delta v_\parallel / v_\perp$ where $\Delta v_\parallel$ is the net acceleration due to those waves near resonance multiplied by the time a typical particle remains in resonance, Δt. So

$$\Delta\alpha \simeq \frac{\Delta v_\parallel}{v_\perp} \simeq \frac{e v_\perp B' \Delta t}{m_e v_\perp} = \Omega^- \frac{B'}{B_0}\Delta t$$

where B' is the wave amplitude near resonance.

Hence the pitch angle diffusion coefficient

$$D \equiv \frac{\langle(\Delta\alpha)^2\rangle}{2\Delta t} \approx \frac{(\Omega^-)^2}{2}\left(\frac{B'}{B_0}\right)^2 \Delta t,$$

where angular brackets denote an average over the fluctuations spectrum (Kennel and Petschek, 1966).

Noting that $\Omega^- = \frac{2\pi}{T^-}$, where T^- is the electron gyroperiod, this reduces to

$$D = \pi\Omega^-\left(\frac{B'}{B_0}\right)^2 f_{\text{res}}, \qquad (6)$$

where f_{res} = fraction of a gyroperiod that the particle remains in resonance.

For strong diffusion, electrons diffuse across the loss cone many times in a $\frac{1}{4}$ bounce period ($\tau_B/4$). Taking the loss cone pitch angle α_L as small,

$$\alpha_L^2 \approx B_{\text{eq}}/B_A,$$

where B_{eq} = magnetic field strength at equator and B_A = magnetic field strength at 100 km altitude conjugate to B_{eq}.

So taking the lower limit on the strong diffusion coefficient as D_{SD}, where

$$D_{SD} = \alpha_L^2/(\tau_B/4) = \frac{4B_{\text{eq}}}{\tau_B B_A},$$

then for strong diffusion:

$$\pi \Omega_{eq}^{-} \left(\frac{B'_{SD}}{B}\right)^2 f_{res} > \frac{4 B_{eq}}{\tau_B B_A} \tag{6a}$$

where B'_{SD} = wave amplitude required for strong diffusion,

$$\text{i.e.} \quad B'_{SD} > \left(\frac{4 B_{eq}^3}{\pi B_A \Omega_{eq}^{-} f_{res} \tau_B}\right)^{1/2}. \tag{7}$$

The fraction f_{res} is unknown, but it is not unreasonable to assume a value that corresponds to the time taken for a particle to change the phase difference between its perpendicular velocity and the magnetic component of the wave by 1 radian (Kennel and Petschek, 1966), i.e. $f_{res} \sim 2$ (see Appendix). Equation (7) can be further simplified by using the relation (Inan, 1977)

$$\sin^2 \alpha_{eq} = \frac{(1+M)^3}{L^2[4L^2 - 3L(1+M)]^{1/2}}$$

where α_{eq} is the pitch angle at the equator of an energetic electron which mirrors at height M in earth radii and crosses the geomagnetic equator at height L in earth radii (the McIlwain L-parameter).

For precipitation $M \ll 1$, in which case $\sin^2 \alpha_L \simeq \frac{1}{2L^3}$, for $L > 5$, i.e. $\alpha_L^2 \simeq \frac{1}{2L^3}$. Hence Eq. (7) can be simplified to

$$B'_{SD} > \frac{B_{eq}}{(\pi L^3 \Omega_{eq}^{-} \tau_B)}. \tag{8}$$

Figure 1 shows B'_{SD} as a function of energy of resonating electrons, using the values of τ_B given by Hamlin et al. (1961), at different L-shells in the vicinity of the GEOS 2 orbit (nominally at $L = 6.5$). The values of Ω_{eq}^{-} used are based on the range of values of B_{eq} on the nightside normally observed by the magnetometer, and approximately centred on the values predicted from modelling (Kosik, 1975). The shaded area shows the expected range of values of B'_{SD} required to put energetic electrons on strong pitch angle diffusion at the GEOS 2 orbit. The lines labelled (a), (b), (c) in the lower part of Fig. 1 refer to previously published field strengths of pulsed hiss in the nightside magnetosphere. (a) is based on the conclusions of Gough and Korth (1982) that when the loss cone is empty (count rate less than 0.4 times the count rate just outside the loss cone) the most likely ELF intensity is 10^{-2} pT.Hz$^{-1/2}$, and that when the loss cone is partially filled in (count rate greater than 0.4 times the count rate just outside the loss cone) the most likely ELF intensity is two orders of magnitude higher, i.e. 1 pT.Hz$^{-1/2}$. These values have been used with an assumed effective bandwidth of 500 Hz to produce the lines labelled (a). (b) corresponds to the background and peak values reported by Tsurutani and Smith (1974) – from OGO 5 observations of post-midnight chorus – of 10^{-1} pT.Hz$^{-1/2}$ and 3.16 $\times 10^{-1}$ pT.Hz$^{-1/2}$ respectively, again integrated over a 500 Hz bandwidth. Although the authors do not distinguish between pulsed hiss and chorus it is clear from their published specimen spectrograms that some of the

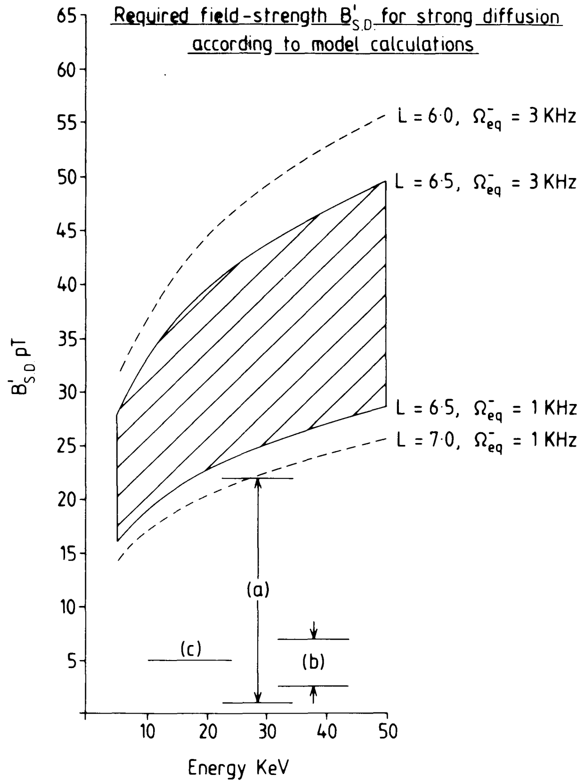

Fig. 1. Comparison of the expected ELF field strength B'_{SD} at geostationary orbit required to put energetic electrons into strong pitch angle diffusion, as a function of the energy of the electrons, with previously reported observed pulsed ELF hiss field strengths by spacecraft. (**a**) is from Gough and Korth (1982), (**b**) is from Tsurutani and Smith (1974), (**c**) is from Gough et al. (1981). The *shaded area* is the expected range of required field strengths for typical nightside electron gyrofrequencies (Ω_{eq}^{-}) encountered by GEOS 2

events defined as chorus by them are spectrally similar to pulsed hiss. (c) is the reported early GEOS 1 measurements (Gough et al, 1981) on pulsed ELF hiss at 00.00 MLT for strong magnetic activity ($Kp > 4$) of 2.1×10^{-1} pT.Hz$^{-1/2}$, again integrated over a 500 Hz bandwidth.

As has been pointed out previously (Thomas, 1983), there appears to be a discrepancy between observed and required field strengths to put energetic electrons into strong pitch angle diffusion, although some previously published experimental results (e.g. (a) in Fig. 1) can now account for pulsating aurora. There does appear to be a difference, however, between the required field strengths of Fig. 1, and the required field strengths presented in Thomas (1983). This is, to some extent, due to the dipole field model used by Thomas to calculate Ω_{eq}^{-}, and hence B_{eq}, but is mainly due to a different interpretation of f_{res}. Whereas f_{res} is defined above as the fraction of a gyroperiod that particle and wave are in resonance, Thomas defines f_{res} as the fraction of a bounce period that the particle spends in "effective resonance".

The assumption that $D_{SD} = \alpha_L^2/(\tau_B/4)$ is used in the above derivation because $\tau_B/4$ provides an estimate of the time required by a full loss cone to empty via precipitation (i.e. empty to the level where the con-

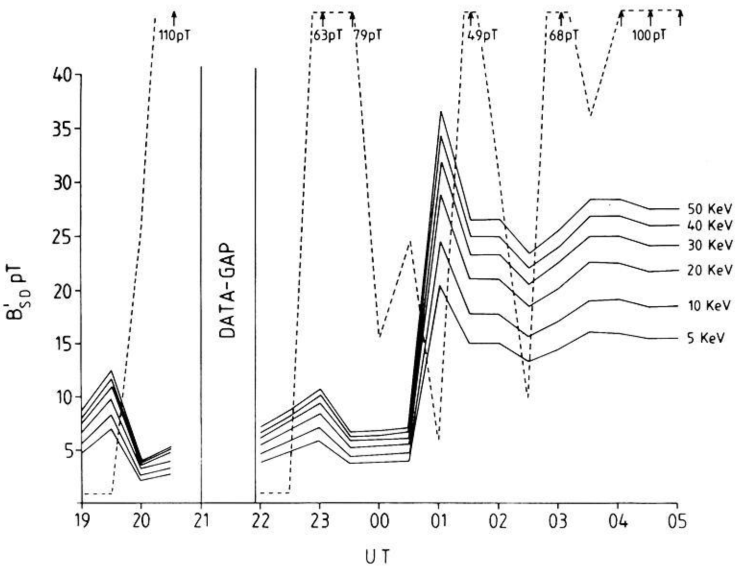

Fig. 2. Required values of B'_{SD} during the night of 7–8 October 1981 for different energy electrons (*solid lines*). The *broken line* is the observed peak field strengths at GEOS 2. (Values have been written in where these go off the scale)

sequent temperature anisotropy becomes unstable to the whistler mode instability). Hence $\alpha_L^2/(\tau_B/4)$ provides a lower limit to the rate of scattering necessary to maintain a full loss cone (strong diffusion). The left hand side of Eq. (6a) describes the behaviour of an individual electron which is limited (if strong diffusion is to be maintained) by the average rate of scattering required to maintain an isotropic pitch angle distribution about the loss cone, which is estimated as $\alpha_L^2/(\tau_B/4)$.

Thomas' formulation implicitly assigns a value of unity to f_{res}, as defined in Eq. (6) above, and assumes that each scattering wave-particle interaction produces a scattering in pitch angle $\geqq \alpha_L$, and that many of these occur during the relatively small fraction (0.1–0.2) of a quarter-bounce period that an electron spends in the interaction region ($\sim \pm 5$–$10°$ of MLT about the geomagnetic equator on auroral L-shells). This is clearly a more stringent requirement than the minimum requirement for near strong diffusion described by Eq. (6a). This results in much greater amplitudes being necessary in the wave-field than are required by the approach taken in this paper.

Case study

To check whether the discrepancy between observed and required field strengths actually occurs during pulsating aurora a night (7–8 October 1981) on which a series of substorms, and several displays of pulsating aurora, occurred has been selected for analysis.

Throughout the period 17.30 to 05.00 the dominant emission type observed on GEOS 2 was pulsing hiss type A (Gibbons and Ward, 1981) where the minima of the pulses are at, or below, the minimum detectable level (25 dB above 1 fT.Hz$^{-1/2}$ for the 200–625 Hz filter – filter 1)) changing to type B (where the minima of the pulsations, and hence the background levels, are well above the minimum detectable level (Gibbons and Ward, 1981)) at 03.10. There is also a data gap from 20.49 to 21.56. Both types of pulsed hiss in general reach similar peak field strengths and so both are equally likely to produce pulsating aurorae. Case studies (Ward et al., 1982) and morphological comparison (Ward, 1983) indicate that type A pulsed hiss is associated with pulsating patches whereas type B pulsed hiss is associated with the recurrent propagating form type of pulsating aurora.

Activity began at 19.45 and persisted (with a few minutes break between 20.43 and 20.45) into the data gap. The activity was pulsed hiss type A reaching peak amplitudes $\sim$80–90 dB above 1 fT · Hz$^{-1/2}$ in the 200–625 Hz filter. There was no activity after the data gap, except for a slight burst between 22.16 and 22.26, until 22.47. Again type A pulsed hiss reached 80–90 dB peak levels. This continued until 23.40, then resumed again at 23.48, with a slightly decreased intensity as well as a decreased pulsation rate, until 00.47. Activity resumed again at 00.56 and continued until the end of the period, again with reduced peak intensities and pulsation rates. The change from type A to type B at 02.10 was accompanied by an increase in the frequency bandwidth at which the maximum activity was observed, corresponding possibly to a decrease in the energy of the particles at which pitch angle anisotropy maximised.

Although the magnetometer on GEOS 2 was not operational, values of the local geomagnetic field strength were obtained from the active plasma sounder experiment S 301 (Jones, 1978) which provides a measurement of the local gyrofrequency. Spot values every half-hour of the required wave field strength for strong diffusion have been calculated from Eq. (7) for different electron energies using the IGS reference field for 1980 for the internal field model and an averaged value of the disturbed and super-disturbed Mead-Fairfield magnetic field models (Mead and Fairfield, 1975) for the external field model, to calculate B_A at 100 km. These are plotted in Fig. 2 where data was available (all instruments on GEOS 2 suffered the same data gap). The sudden surge in B'_{SD} between 00.30 and 01.00 is due to a dramatic leap in the value of the local gyrofrequency from 0.611 kHz to 3.217 kHz and then remaining between 2 and 3 kHz for the remainder of the period. All other variations in B'_{SD} are also due to variations in B_{eq}, as the calculated variation in the value of B_A were less than 0.25%.

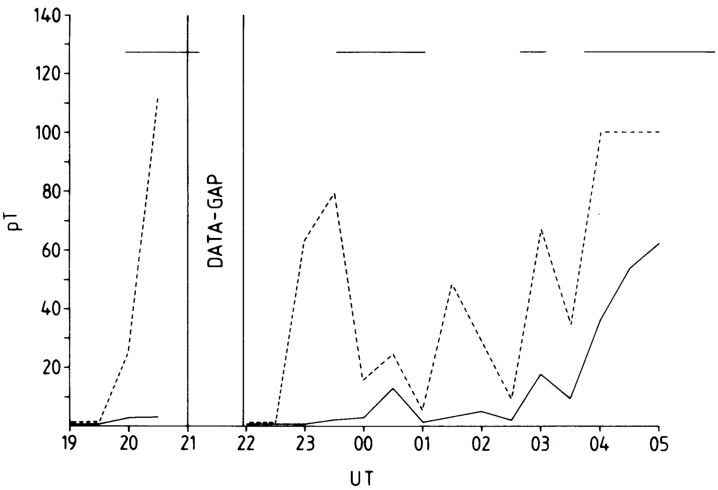

Fig. 3. Observed peak (*broken line*) and average (*solid line*) field strengths of pulsed ELF hiss at GEOS 2 during the night of the 7/8 October 1981. The *horizontal lines* in the upper half of the figure correspond to the times of substorms as indicated by magnetograms (*H*-component) from Tromso (courtesy of W. Stuart and T. Harris, British Geological Survey, Edinburgh)

In Fig. 3 are plotted spot values coincident with the times chosen for Fig. 2 of the observed ELF/VLF wave field strength as observed by the filter bank on GEOS 2. The lower solid line is the locus of the average observed field strengths over 1 min every half-hour and the upper broken line is the locus of peak field strengths observed within ± 2 min of the selected time (every half-hour). The bandwidths used correspond to either half the observed gyrofrequency or a calculated bandwidth based on the ratio of the signals in the two adjacent filters where the signal strength was greatest (usually the 0.2–0.625 kHz and the 0.625–1.25 kHz filters) and calculated according to the transfer functions of the two filters to white noise, whichever was the smaller.

Comparing Figs. 2 and 3 it is clear that there are several periods when the observed maximum field strengths are sufficient to put energetic particles into strong pitch angle diffusion, and since the observed emission-type throughout this period was pulsed hiss, these should correspond to the occurrence of pulsating aurora at the foot of the GEOS field line. These periods are from approx. 19.45 UT to at least 20.30 UT and possibly beyond, from approx. 22.45 UT to 00.45 UT, from approx. 01.15 UT to approx. 02.15 UT, and from approx. 02.45 UT to 05.00 UT. The broken line (maximum field strengths observed) of Fig. 3 has been transferred on to Fig. 2 to show these periods. Times are only approximate as the values used are spot measurements every half-hour. Hence, the start and stop times quoted above have been taken as halfway between two consecutive measurements, unless the times of sufficient field strength coincided with start and stop times of activity, e.g. at 22.47, in which case a more accurate timing to ± 1 min can be used.

Auroral observations (P. Rothwell [personal communication]) indicated at least two periods in the early evening to midnight sector when pulsating aurora occurred after two break-ups. The first break-up occurred at 19.46–19.48 UT to the far south of the normally expected position for the GEOS footprint, pulsating aurorae accompanying this post break-up phase were observed until 20.20 when cloud spoiled the viewing conditions. The second break-up occurred at 23.30, but again cloud cover spoiled the viewing conditions, however pulsating aurorae were also observed to occur during this post break-up phase. There were further substorms during the night (see Fig. 3) and associated pulsating aurorae. However, precise timing of pulsating auroral activity after midnight are not available because of the cloudy conditions.

Summary

Over several extended periods during the night of 7–8 October 1981 the peak field strengths of pulsed ELF hiss observed on GEOS 2 were sufficient to put energetic electrons into strong pitch angle diffusion. Pulsating auroral activity conjugate to GEOS 2 was also reported during at least some of these intervals. It does appear, however, that a field strength that would put electrons into strong diffusion is a necessary, but not sufficient, condition for pulsating aurora to occur. Pulsed hiss at GEOS 2 is not always associated with pulsating aurora, although the converse is true (Thomas, 1982). Previous observations have shown that the peak strength of pulsed ELF hiss is more dependent upon the degree of temperature anisotropy in the energetic electrons, rather than upon the total flux (Ward, 1983), whereas the net flux of energetic particles parallel to the geomagnetic field direction is the important factor in generating visible auroral displays. This is possibly the reason why pulsed ELF hiss of sufficient intensity is not always associated with pulsating aurora.

Conclusions

Observations and theoretical arguments indicate that the energetic electron population which is the source of the precipitating particles must be in a state of strong pitch angle diffusion during pulsating aurora. Comparisons of previous observations of pulsed ELF hiss field strengths with the calculated required field strength for strong diffusion in a dipole field model (i.e. assuming the loss cone angle is a function of L only), and for typically observed geomagnetic field strengths on the nightside at geostationary orbits, imply that the amplitude of pulsed ELF hiss is not sufficient to put energetic electrons into strong pitch angle diffusion. However, in situ coincident measurements of the geomagnetic field and ELF pulsed hiss amplitudes and bandwidths by GEOS 2 during a series of substorms reveal that there are periods when the peak ELF field strengths are sufficient to provide strong pitch angle diffusion. During at least parts of these periods pulsating aurorae were occurring in the vicinity of the footprint of the GEOS 2 field line.

Appendix

Estimating f_{res}

When the wave and particle, travelling in opposite directions, have phase and parallel velocities respectively such that $kV_{\parallel} = \omega - \Omega^-$, the particle feels a constant magnetic field, from the magnetic component of the wave, normal to its circular motion about the background magnetic field. It thus suffers an acceleration along the magnetic field and hence a change in $V_{\parallel}$. Eventually, the wave and particle are no longer in Doppler-shifted cyclotron resonance, as the condition $kV_{\parallel} = \omega - \Omega^-$ is no longer satisfied. However the change in $V_{\parallel}$ will cause the particle to be able to resonate with a wave of a different frequency. Also, if the source of the waves were such that the phases of the different frequency waves matched the power spectrum in such a way that the particle always remained in phase with the wave whose Doppler-shifted frequency matched the particle's cyclotron frequency there would be a coherent wave-particle interaction involving diffusion in both pitch angle and energy space. However, for natural sources of broad-band ELF/VLF unstructured noise (hiss) there is no such coherence between waves of different frequencies, the wave-particle interaction is incoherent and there is no diffusion in energy space, only in pitch angle space. In this case the particle and wave resonate for only a short time (short compared with the bounce period of the particle) as they are quickly put out of phase by the consequent acceleration of the particle parallel to the background magnetic field. To estimate the interaction time Δt between a wave and a particle (and hence f_{res}, the fraction of a gyroperiod the particle and wave are in resonance) we shall assume that during the interaction the particle suffers a constant magnetic force from the wave and calculate Δt required to accelerate the particle to such a velocity $V_{\parallel}$ that it is in Doppler-shifted cyclotron resonance with a different frequency wave whose phase difference with the first wave has increased by $\pi/4$ radians in Δt secs., i.e. such that the original wave and the particle have become $\pi/4$ radians out of phase, i.e.

$$(\omega_1 - \omega_2)\Delta t = \pi/4 \tag{9}$$

where ω_1 is the frequency of the wave that, after Δt seconds, has accelerated the particle to such a velocity that it is now in resonance with a wave of frequency ω_2. Assuming the cold plasma approximation for the refractive index and the Doppler-shifted cyclotron resonance condition yields

$$\left[\frac{\omega_2(\omega_2 - \Omega)}{\omega_1(\omega_1 - \Omega)}\right]^{1/2} = \frac{V_{\parallel 2}}{V_{\parallel 1}}, \tag{10}$$

where

$$k_1 V_{\parallel 1} = \omega_1 - \Omega$$

and

$$k_2 V_{\parallel 2} = \omega_2 - \Omega$$

and

$$V_{\parallel 2} = V_{\parallel 1} + \left(\frac{\Omega B'}{B_0} \cdot V_{\perp} \cdot \Delta t\right). \tag{11}$$

B' is the amplitude of the magnetic component of the wave frequency ω_1, B_0 and Ω are the background magnetic field and gyrofrequency respectively, and $V_{\parallel}$, $V_{\perp}$ are the velocities of the particle parallel and perpendicular to B_0 respectively.

For numerical simplicity, since we are only seeking an estimate for f_{res}, assume an initial 45° pitch angle such that $V_{\parallel 1} = V_{\perp}$ and typical magnetospheric conditions at the GEOS 2 orbit on the nightside (i.e. $\omega_1/2\pi \sim 1$ kHz, $\Omega \sim 3$ kHz, $B' \sim 50$ pT) then substitution from (i) and (iii) into (ii) yields a cubic in Δt with a smallest solution of $\Delta t \simeq 0.595$ ms, i.e. $f_{res} = 1.78$. Hence the assumption of $f_{res} = 2$ as the time taken for a particle to change the phase difference between its perpendicular velocity and the magnetic component of the wave by 1 radian is seen to be a reasonable estimate.

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