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Solution of the stationary approximation for MT fields in the layered Earth with 3D and 2D inhomogeneities

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Abstract. The solution of the stationary approximation for MT fields in the layered Earth with a 3D or 2D disturbing body can be effectively performed by means of generalized double-layer potential. This scalar potential expresses the effect of current dipoles distributed over the surface of the disturbing body. The density of the double layer (dipole layer) can be obtained by solution of the scalar integral equation of the Fredholm type with a weakly singular kernel. Formulae and results of numerical computations of the intensity of electric and magnetic fields on the surface of the Earth for some block models are presented. The anomalous field is similar to the field of a horizontal electric current dipole oriented in the direction of the exciting electric field and situated in the disturbing body. This method gives general features of the disturbing field for long periods of MT fields and is also applicable to geothermal potential problems.

Key words: Electromagnetic induction – Conductivity anomalies – Electrical methods – Potential fields – Geothermal anomalies

Introduction

In the last decade considerable progress has been achieved in numerical modelling of MT fields for media with 3D inhomogeneities of arbitrary shape. The most advantageous method for the solution is the method of vector integral equations, which was presented and developed, e.g., in Raiche (1974), Weidelt (1975), Stodt et al. (1981) and Hvoždara (1981a, b). First the basis of the theory is given briefly and then the useful long-period (stationary) approximation for the MT field is presented. Consider a 3D disturbing body embedded in the L-th layer of the layered Earth. The conductivity of the body is σ_T, its volume τ and surface S; the body is assumed to be fully embedded in the L-th layer , without penetration into neighbouring layers. The layered Earth has planar boundaries $z=0, h_1, h_2, \ldots, h_{N-1}$ and the electrical conductivity in the m-th layer is σ_m; we assume all layers and the disturbing body to be nonmagnetic. The air in the upper halfspace $z<0$ is considered non-conductive and non-magnetic. The undisturbed (primary) electric field we denote by $\mathbf{E}^p(\mathbf{r})$ and which is assumed to be have time dependence $\exp(-i\omega t)$, where $\omega=2\pi/T$ is the angular frequency. We observe the electric field $\mathbf{E}(\mathbf{r})$ at the point $P(\mathbf{r})$, which is different from the primary one, due to the presence of the disturbing body. The basic vector equation for the $\mathbf{E}(\mathbf{r})$ field is (Raiche, 1974):

$$\mathbf{E}(\mathbf{r})=\mathbf{E}^p(\mathbf{r})+(\sigma_T-\sigma_L)\int_\tau \bar{\bar{\Gamma}}(\mathbf{r},\mathbf{r}')\,\mathbf{E}(\mathbf{r}')\,d\tau', \tag{1}$$

where $\bar{\bar{\Gamma}}(\mathbf{r},\mathbf{r}')$ is Green's tensor function (dyadic), the element $\Gamma_{ij}(\mathbf{r},\mathbf{r}')$ being calculated as the i-th component of the electric field at a point $P(\mathbf{r})$ due to the elementary j-oriented electric dipole located at point $Q(\mathbf{r}')$. The elements $\Gamma_{ij}(\mathbf{r},\mathbf{r}')$ of the tensor satisfy the inhomogeneous Helmholtz equation:

$$\nabla^2\Gamma_{ij}+k^2\Gamma_{ij}=-i\omega\mu_0\,\delta(\mathbf{r}-\mathbf{r}')\,\delta_{ij}, \tag{2}$$

$i,\ j=1,2,3,\ (x,y,z),\ k=(1+i)(\omega\sigma\mu_0/2)^{1/2}$ is the wave number. $\delta(\mathbf{r}-\mathbf{r}')$ is Dirac's function and δ_{ij} is Kronecker's symbol. These functions satisfy well known continuity conditions on planar boundaries $z=0, h_1, h_2, \ldots$ of the layered medium. Details of the method for calculating Green's dyadic function are presented, e.g., in Raiche (1974), Hvoždara (1981a). It is proved that the components of the dyadic are:

$$\Gamma_{ij}(\mathbf{r},\mathbf{r}')=i\omega\mu_0\,{}^j\mathbf{F}\,\delta_{ij}+\sigma^{-1}\frac{\partial}{\partial x_i}\operatorname{div}{}^j\mathbf{F}. \tag{3}$$

The vector potential of the j-th oriented dipole ${}^j\mathbf{F}$ has horizontal and vertical components for the case of x- and y-oriented dipoles ($j=1,\ 2\equiv x,\ y$) and a vertical ($z-$) component only in the case of a vertical dipole ($j=3\equiv z$). The component $i=j$ of the ${}^j\mathbf{F}$ contains the source term

$$\exp[ik(\mathbf{r}-\mathbf{r})]\cdot|\mathbf{r}-\mathbf{r}'|^{-1}\cdot(4\pi)^{-1}.$$

If we consider the point $P(\mathbf{r})$ to be in the volume τ of the disturbing body, then Eq. (1) is a vector integral equation with a dyadic kernel which is weakly singular. This singularity is integrable by the method given by Weidelt (1975). According to experience, it is suitable, in the numerical solution of the integral equation, Eq. (1), to divide the volume τ into cubic elements in order to decrease numerical errors. After solution of the integral equation, the vector integral Eq. (1) can be used for the calculation of the electric field outside the body.

A similar volume integral formula holds for the magnetic field too:

$$\mathbf{H}(\mathbf{r}) = \mathbf{H}^p(\mathbf{r}) + (\sigma_T - \sigma_L) \int_\tau \bar{\bar{h}}(\mathbf{r}, \mathbf{r}') \, \mathbf{E}(\mathbf{r}') \, d\tau', \tag{4}$$

where $\mathbf{H}^p(\mathbf{r})$ is the primary magnetic field corresponding to $\mathbf{E}^p(\mathbf{r})$, $\bar{\bar{h}}(\mathbf{r}, \mathbf{r}')$ is the dyadic of the magnetic effect of the dipoles induced in the disturbing body,

$$h_{ij}(\mathbf{r}, \mathbf{r}') = (\operatorname{curl} {}^j\mathbf{F})_i. \tag{5}$$

Numerical calculations by the vector integral equation method is rather computer time consuming, because we have to deal with a system of three scalar integral equations of Fredholm type for components E_x, E_y and E_z. Both $\Gamma_{ij}(\mathbf{r}, \mathbf{r}')$ and $E_i(\mathbf{r})$ are complex numbers, which increases demands on computer memory and time. Nevertheless, calculations were performed by this method for some cases of prismatic bodies embedded in a halfspace or in a two-layered Earth. They show that the disturbing EM field is like the EM field of a horizontal electric dipole oriented in the direction of the inducing electric field and situated near the centre of the body.

Long-period asymptotics

In solving EM problems it is often very useful to know long-period asymptotics, which in our problem, is equivalent to the stationary current approximation. We can obtain this approximation by putting the angular frequency ω to zero in time-harmonic formulae, i.e. $k^2 \to 0$. For this case we have, according to Eq. (3):

$$\Gamma_{ij}(\mathbf{r}, \mathbf{r}') = \sigma^{-1} \frac{\partial}{\partial x_i} \operatorname{div} {}^j\mathbf{F}.$$

It can be proved that vector potentials ${}^j\mathbf{F}$ have the following form in the stationary case:

$${}^j\mathbf{F} = (4\pi)^{-1} \{ |\mathbf{r} - \mathbf{r}'|^{-1} \mathbf{e}_j + \partial \phi / \partial x_j' \mathbf{e}_z \}, \tag{6}$$

$j = 1, 2, 3 \equiv (x, y, z)$, where ϕ is a harmonic function satisfying pertinent boundary conditions imposed on vector potentials ${}^j\mathbf{F}$: e.g. for the halfspace

$$\phi = \ln[R_+ + (z + z')],$$

where

$$R_+ = [(x - x')^2 + (y - y')^2 + (z + z')^2]^{1/2}.$$

We find that each element of the Green's dyadic $\bar{\bar{\Gamma}}(\mathbf{r}, \mathbf{r}')$ can be derived from a single scalar Green's function $g(\mathbf{r}, \mathbf{r}')$, which corresponds to the point current source in the L-th layer. Similarly, as in Hvoždara (1983a), we obtain

$$\Gamma_{ij}(\mathbf{r}, \mathbf{r}') = -(4\pi\sigma_L)^{-1} \frac{\partial^2}{\partial x_i \partial x_j'} g(\mathbf{r}, \mathbf{r}'), \tag{7}$$

and Eq. (1) for the i-th component gives:

$$E_i(\mathbf{r}) = E_i^p(\mathbf{r}) - \frac{\sigma_T - \sigma_L}{4\pi\sigma_L} \frac{\partial}{\partial x_i} \int_\tau E_j(\mathbf{r}') \frac{\partial}{\partial x_j'} g(\mathbf{r}, \mathbf{r}') \, d\tau'. \tag{8}$$

The scalar Green's functions $g_m(\mathbf{r}, \mathbf{r}')$ satisfy the equations:

$$\nabla^2 g_m(\mathbf{r}, \mathbf{r}') = \begin{cases} 0, & m \neq L \\ -4\pi\delta(\mathbf{r} - \mathbf{r}'), & m = L, \end{cases} \tag{9}$$

and the known boundary conditions for the scalar potential of the stationary point current source embedded in the point $Q(\mathbf{r}') \in \tau$ of the L-th layer; index m specifies the Green's function $g(\mathbf{r}, \mathbf{r}')$ for the individual layers where point P is considered.

It is well known that the stationary electric field can be obtained from the gradient of the scalar potential $U(\mathbf{r})$, i.e.:

$$E_i = -\frac{\partial U(\mathbf{r})}{\partial x_i}, \quad E_j(\mathbf{r}') = -\frac{\partial U_T(\mathbf{r}')}{\partial x_j'}, \quad E_i^p = -\frac{\partial V(\mathbf{r})}{\partial x_i}, \quad Q(\mathbf{r}') \in \tau, \tag{10}$$

$U_T(\mathbf{r})$ denotes potential inside the body τ and $V(\mathbf{r})$ is the potential of the primary electric field. Equation (8) then gives:

$$U(\mathbf{r}) = V(\mathbf{r}) + \frac{1 - \sigma_T/\sigma_L}{4\pi} \int_\tau \operatorname{grad}' g(\mathbf{r}, \mathbf{r}') \cdot \operatorname{grad}' U_T(\mathbf{r}') \, d\tau'. \tag{11}$$

Now we can apply Green's formula for the volume integral containing the scalar product of two gradients, Van Bladel (1964):

$$\int_\tau \operatorname{grad}' A \cdot \operatorname{grad}' B \, d\tau' = \int_S A \, \partial B / \partial n' \, dS' - \int_\tau A \nabla^2 B \, d\tau', \tag{12}$$

where $\partial B / \partial n' = \mathbf{n}' \cdot \operatorname{grad}' B$ is the derivative with respect to the outward pointing normal $\mathbf{n}'$. We put $A = U_T(\mathbf{r}')$, $B = g(\mathbf{r}, \mathbf{r}')$ and with respect to Eq. (9) the potential outside the body τ is given by:

$$U_m(\mathbf{r}) = V_m(\mathbf{r}) + \frac{1 - \sigma_T/\sigma_L}{4\pi} \cdot \int_S U_T(\mathbf{r}') \frac{\partial}{\partial n'} g_m(\mathbf{r}, \mathbf{r}') \, dS', \quad P(\mathbf{r}) \notin \tau. \tag{13}$$

If we consider the point $P(\mathbf{r}) \in \tau$, we have to take into account that $\nabla^2 g_L(\mathbf{r}, \mathbf{r}') = -4\pi\delta(\mathbf{r} - \mathbf{r}')$, for $m = L$, and therefore:

$$\int_\tau U_T(\mathbf{r}') \nabla^2 g_L(\mathbf{r}, \mathbf{r}') \, d\tau' = -4\pi \int_\tau U_T(\mathbf{r}') \delta(\mathbf{r} - \mathbf{r}') \, d\tau' = -4\pi U_T(\mathbf{r}).$$

Then we obtain, for $P(\mathbf{r}) \in \tau$:

$$U_T(\mathbf{r}) = (\sigma_L/\sigma_T) \left\{ V_L(\mathbf{r}) + \frac{1 - \sigma_T/\sigma_L}{4\pi} \cdot \int_S U_T(\mathbf{r}') \frac{\partial}{\partial n'} g_L(\mathbf{r}, \mathbf{r}') \, dS' \right\}. \tag{14}$$

In this way, we have proved that long-period asymptotics of the time-harmonic fields give the same formulae as were obtained by another method (in Hvoždara, 1982; 1983) – using Green's theorems and pertinent boundary conditions for the electrical potential fields. Both Eqs. (13) and (14) show us that the potential of the anomalous electric field is given by a surface integral, which is, in principle, the generalized dipole-layer (double-layer) potential – instead of the classical kernel $\partial|\mathbf{r}-\mathbf{r}'|^{-1}/\partial n'$, we have $\partial g(\mathbf{r},\mathbf{r}')/\partial n'$. Each surface element dS' at the point $Q(\mathbf{r}')$ of the boundary S possesses a current dipole oriented in the direction normal $\mathbf{n}'$ and of moment $dM(\mathbf{r}')=(\sigma_L-\sigma_T)\,U_T(\mathbf{r}')\,dS'$. The anomalous potential is a superposition of potentials due to all dipoles distributed over the surface S of the disturbing body. It is advantageous that, in contrast to time-harmonic problems, we deal with surface integrals of scalar functions, which saves a lot of computer time and memory.

The Green's function for the simplest case (a disturbing body surrounded by an unbounded conductive medium) is well known:

$$g(\mathbf{r},\mathbf{r}')=R^{-1}=|\mathbf{r}-\mathbf{r}'|^{-1}=[(x-x')^2+(y-y')^2+(z-z')^2]^{-1/2} \qquad (15)$$

and was used for electrostatic problems, e.g. Van Bladel (1964), and for elastostatic problems by Jaswon and Symm (1977).

For the case of a halfspace ($N=L=1$), we have Green's function in the form:

$$g(\mathbf{r},\mathbf{r}')=R^{-1}+R_+^{-1}, \qquad (16)$$

where $R_+=[(x-x')^2+(y-y')^2+(z+z')^2]^{1/2}$ is the harmonic term which expresses the mirror effect of the halfspace surface. We use this function in numerical calculations for a block body embedded in a halfspace. Results are given in Hvoždara (1983a).

For the case of a two-layered Earth with a 3D body in the substratum we have derived Green's function in the form:

$$g_1(\mathbf{r},\mathbf{r}')=2(1+\sigma_1/\sigma_2)^{-1}\cdot\sum_{n=0}^{\infty}(-q)^n\{[\rho^2+(2nh+z-z')^2]^{-1/2}+[\rho^2+(2nh+z+z')^2]^{-1/2}\}, \qquad z\in\langle 0,h\rangle, \qquad (17)$$

and

$$g_2(\mathbf{r},\mathbf{r}')=R^{-1}+q[\rho^2+(z+z'-2h)^2]^{-1/2}+(1-q^2)\sum_{n=0}^{\infty}(-q)^n[\rho^2+(2nh+z+z')^2]^{-1/2}, \qquad z>h, \quad (18)$$

where

$$\rho^2=(x-x')^2+(y-y')^2, \qquad q=(1-\sigma_1/\sigma_2)/(1+\sigma_1/\sigma_2),$$

σ_1 and σ_2 are electrical conductivities of the layer $z\in\langle 0,h\rangle$ and substratum $z>h$, respectively. In a similar way we can obtain the necessary scalar Green's functions for other models of the layered Earth.

For calculation of the potential by Eqs. (13) and (14), it is necessary to know the distribution of the potential $U_T(\mathbf{r}')$ over the surface S. It can be calculated by means of an integral equation, which we obtain in the limit $P(\mathbf{r})\to S_-$ (from the interior) in Eq. (14), and considering the discontinuous properties of the double-layer potential on the surface S. Then we obtain the following integral equation with weakly singular kernel:

$$U_T(\mathbf{r})=\frac{2}{1+\sigma_T/\sigma_L}\,V_L(\mathbf{r})+\frac{\beta}{2\pi}\cdot ⨍_S U_T(\mathbf{r}')\,\partial g_L(\mathbf{r},\mathbf{r}')/\partial n'\,dS', \qquad P(\mathbf{r})\in S, \qquad (19)$$

where $\beta=(1-\sigma_T/\sigma_L)/(1+\sigma_T/\sigma_L)$. The integral on the RHS of Eq. (19), denoted by $⨍$, is understood in the sense of the principal value; it means that a small area around the point $P(\mathbf{r})\in S$ is excluded from integration with the singular part $|\mathbf{r}-\mathbf{r}'|^{-1}$ of $g_L(\mathbf{r},\mathbf{r}')$ because this integrable singularity has been taken into account in the limit $P(\mathbf{r})\to S_-$. The harmonic part of $g_L(\mathbf{r},\mathbf{r}')$ is integrated over all the surface S. It is convenient to define the function $f(\mathbf{r})$, $P(\mathbf{r})\in S$ in the form:

$$f(\mathbf{r})=(1-\sigma_T/\sigma_L)[U_T(\mathbf{r})-q_0], \qquad P(\mathbf{r})\in S, \qquad (20)$$

where $q_0=S^{-1}\int_S V_L(\mathbf{r})\,dS$ is the mean value of the exciting (primary) potential on the surface S. Using this function we can rewrite Eqs. (13), (14) and (19) in the form:

$$U_m(\mathbf{r})=V_m(\mathbf{r})+(4\pi)^{-1}\cdot\int_S f(r')\,\partial g_m(\mathbf{r},\mathbf{r}')/\partial n'\,dS', \qquad P(\mathbf{r})\notin\tau, \qquad (21)$$

$$U_T(\mathbf{r})=(\sigma_L/\sigma_T)\{V_L(\mathbf{r})-q_0+(4\pi)^{-1}\cdot\int_S f(\mathbf{r}')\,\partial g_L(\mathbf{r},\mathbf{r}')/\partial n'\,dS'\}+q_0, \qquad P(\mathbf{r})\in\tau. \qquad (22)$$

$$f(\mathbf{r})=2\beta[V_L(\mathbf{r})-q_0]+(\beta/2\pi)\cdot ⨍_S f(\mathbf{r}')\,\partial g_L(\mathbf{r},\mathbf{r}')/\partial n'\,dS', \qquad P\in S. \qquad (23)$$

Since $|\beta|<1$, the integral equation, Eq. (23), can be solved by an iteration method, as performed by Hvoždara (1983a) for a block body in a halfspace.

After solving the integral equation we can calculate potentials outside or inside the disturbing body, using Eqs. (21) and (22). The intensity of the electric field can be calculated as the negative gradient with respect to unprimed coordinates:

$$\mathbf{E}(\mathbf{r})=-\operatorname{grad}U_m(\mathbf{r})=\mathbf{E}_m^p(\mathbf{r})+\mathbf{E}_m^*(\mathbf{r}), \qquad (24)$$

where

$$\mathbf{E}_m^*(\mathbf{r})=-(4\pi)^{-1}\cdot\int_S f(\mathbf{r}')\operatorname{grad}[\mathbf{n}'\cdot\operatorname{grad}'_m g(\mathbf{r},\mathbf{r}')]\,dS'$$

is the anomalous electric field in the m-th layer. It is clear that a magnetic field anomaly $\mathbf{H}_m^*$ corresponds to the anomalous electric current density $\mathbf{j}_m^*=\sigma_m\mathbf{E}_m^*$, according to Maxwell's equations:

$$\operatorname{curl}\mathbf{H}_m^*=\sigma_m\mathbf{E}_m^*, \qquad \operatorname{div}\mathbf{H}_m^*=0. \qquad (25)$$

The formula for the vector $\mathbf{H}_m^*$ can be obtained in the limit $\omega\to 0$ in the volume integral equation, Eq. (4). Using property (6) of the vector potentials and Green's theorems, we can reduce the volume integral to the surface one:

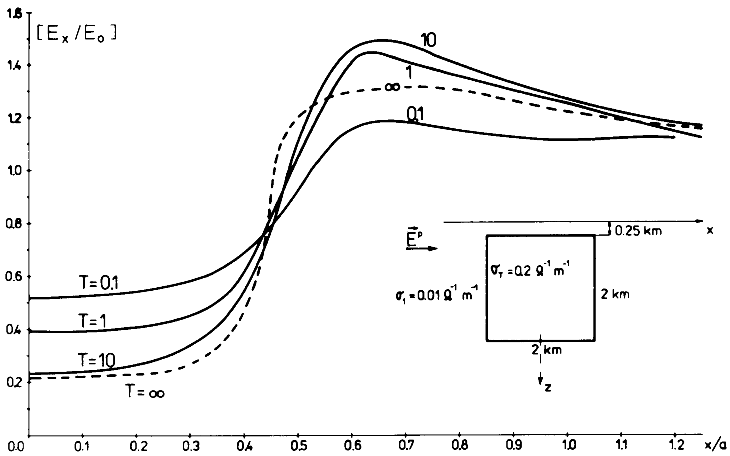

Fig. 1. Curves E_x/E_0 at $z=0$, $y=0$ for 3D block $a=2$ km, $b=1$ km, $c=2$ km, $h_T=1.25$ km, $\sigma_T/\sigma_1=20$ in a time-harmonic plane EM wave, incident on the surface of the halfspace; $\mathbf{E}^p$ is x-oriented, E_0 is undisturbed electric field on the surface of the halfspace

$$\mathbf{H}_m^* = \sigma_L \int_S f(\mathbf{r}') \sum_{j=1}^{3} \operatorname{curl}^j \mathbf{F}(\mathbf{r},\mathbf{r}')\, n_j'\, dS'. \tag{26}$$

In a previous paper (Hvoždara, 1983a) formulae are given for calculating the magnetic field $\mathbf{H}_m^*$ as the superposition of the magnetic fields due to elementary dipoles distributed over the surface S. For geophysical purposes we usually need to know the magnetic field on the surface $z=0$ of the Earth. For this plane, the quite simple formula

$$(H_1^*)_i = \sigma_L (4\pi)^{-1} \int_S f(\mathbf{r}') [h_{ix}(\mathbf{r},\mathbf{r}')\, n_x' + h_{iy}(\mathbf{r},\mathbf{r}')\, n_y']\, dS',$$
$$i = x, y, z, \qquad P(\mathbf{r}) \in (z=0). \tag{27}$$

Here were obtained the following components of the tensor of the magnetic effect due to x- and y-oriented dipoles have been introduced:

$$h_{xx} = (x-x')(y-y')(2R_0+z')R_0^{-3}(R_0+z')^{-2},$$
$$h_{yx} = z' R_0^{-3} - R_0^{-1}(R_0+z')^{-1} + (x-x')^2(2R_0+z')R_0^{-3}(R_0+z')^{-2},$$
$$h_{zx} = (y-y')R_0^{-3},$$
$$h_{yy} = -h_{xx}, \tag{27a}$$
$$h_{xy} = z' R_0^{-3} - R_0^{-1}(R_0+z')^{-1} + (y-y')^2(2R_0+z')R_0^{-3}(R_0+z')^{-2},$$
$$h_{zy} = -(x-x')R_0^{-3},$$
$$R_0 = [(x-x')^2 + (y-y')^2 + (z')^2]^{1/2}.$$

It is interesting that the magnetic field $\mathbf{H}_1^*$ on the surface $z=0$ is independent of z-directed dipoles on the surface S, so that we do not have terms $h_{iz}(\mathbf{r},\mathbf{r}')\, n_z'$ in Eq. (27). The tensor components, Eq. (27a), are independent of the layer parameters σ_m, h_m. The total anomalous magnetic field on the surface $z=0$ depends on these parameters via the density of the dipole layer $f(\mathbf{r}')$.

These formulae and properties can also be useful in the theory and practice of the magnetometric resistivity method (MMR), which was developed by Edwards et al. (eg. Edwards and Howell, 1976). Note that the possibility of calculating the anomalous magnetic field is an advantage of the double-layer potential method, in contrast to the single-layer method which is also used for solving stationary current problems (e.g. Okabe, 1981) but gives no formulae for the magnetic field.

The derived formulae for the calculation of potentials can be easily adopted for two-dimensional (2D) bodies within 2D exciting fields. Let the surface lines of the 2D body be parallel to the y-axis, so that a vertical section s_T of the body in the (x, z)-plane is bounded by a closed curve C. If the potential of the primary electric field is independent of the y-coordinate, the disturbing potential and the density of the double-layer $f(\mathbf{r}')$ will be independent of y' too. The potential at the point $P \equiv (x, z)$ (we put $y=0$) can be calculated by integration in Eqs. (21)–(23):

$$\int_S \ldots dS' = \int_C \int_{-\infty}^{+\infty} \ldots dy'\, dl_Q$$

with respect to coordinate y' of the source point $Q(x', y', z')$. The functions to be integrated are the Green's functions in Eqs. (21)–(23). Then we obtain formulae similar to the 3D ones, but instead of terms of the form $[(x-x')^2+(y-y')^2+(z\pm z')^2]^{-1/2}$ we will have $\ln[(x-x')^2+(z\pm z')^2]^{-1/2}$ and instead of 4π, 2π in the 3D formulae we now have factors 2π, π. The two-dimensional analogy for our basic formulae, Eqs. (21)–(23), will be:

$$U_m(P) = V_m(P) + (2\pi)^{-1} \cdot \int_C w(Q)\, \partial \bar{g}_m(P,Q)/\partial n_Q\, dl_Q, \qquad P \notin s_T \tag{28}$$

$$U_T(P) = (\sigma_L/\sigma_T)\{V_L(P) - v_0 + (2\pi)^{-1}\} \cdot \int_C w(Q)\, \partial \bar{g}_L(P,Q)/\partial n_Q\, dl_Q + v_0, \qquad P \in S_T \tag{29}$$

$$w(P) = 2\beta[V_L(P) - v_0] + (\beta/\pi) \oint_C w(Q)\, \partial \bar{g}_L(P,Q)/\partial n_Q\, dl_Q, \qquad P \in C, \tag{30}$$

where $v_0 = C^{-1} \int_C V_L(P)\, dl_P$ is the mean value of the primary (y-independent) potential on the boundary curve C; $\mathbf{n}_Q \equiv (n_x, n_z)$ is the outward normal at point

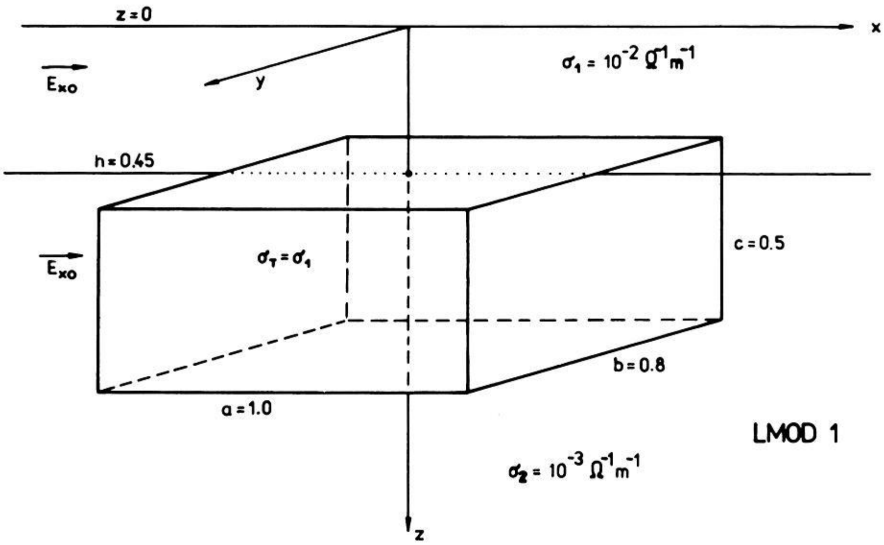

Fig. 2. Model of a 3D (block-like) depression in the two-layered Earth induced by uniform, stationary, x-oriented telluric field

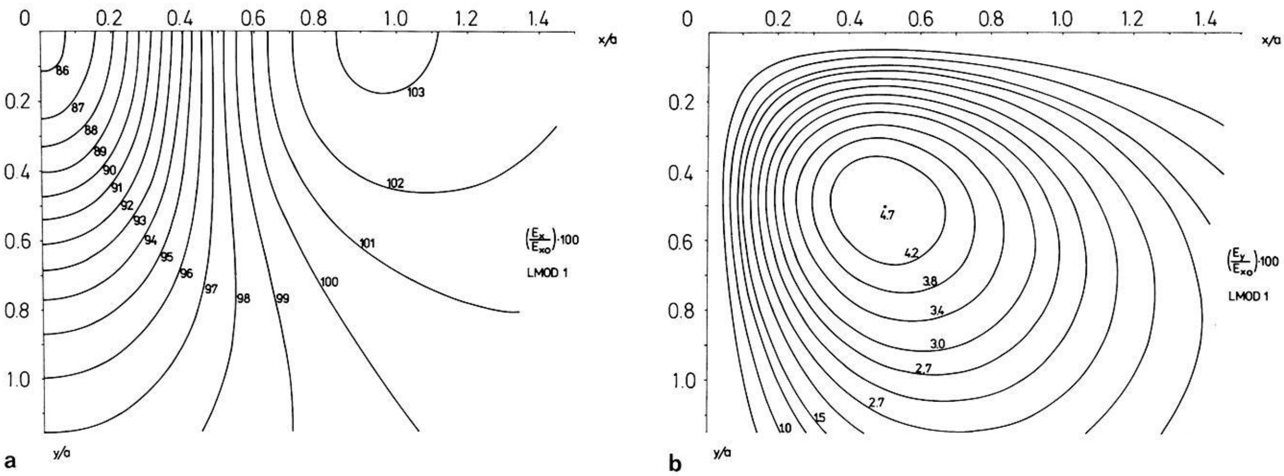

Fig. 3a and b. Isolines of E_x/E_{x0}, E_y/E_{x0} (multiplied by 100) on the surface $z=0$ in the quadrant $x \geqq 0$, $y \geqq 0$. In other quadrants E_x is symmetrical, E_y changes its sign like sign $(x \cdot y)$

$Q \equiv (x', z') \in C$, $w(Q)$ is the dipole-layer density, which must be determined by the integral equation, Eq. (30). The functions $\bar{g}_m(P, Q)$ are two-dimensional Green's functions – we can obtain them from 3D Green's functions $g_m(\mathbf{r}, \mathbf{r}')$ by integration with respect to y' from $-\infty$ to $+\infty$ and putting $y=0$. In the function $\bar{g}_L(P, Q)$ we have logarithmic singularity $\ln[(x-x')^2+(z-z')^2]^{-1/2}$; otherwise $\bar{g}_m(P, Q)$, $m \neq L$, are bounded harmonic functions.

On the basis of the mathematical similarity of potential problems, we can state that the theory of integral formulae with double-layer potentials presented here is applicable not only to geoelectric potential problems, but to similar potential problems of magnetometry and geothermics. Some examples are presented in Hvoždara (1983b), Hvoždara and Schlosser (1983).

The solution for the disturbance of the stationary geothermal field due to the presence of a disturbing body with heat conductivity $\lambda_T \neq \lambda_L$ is similar to the geoelectrical problem, but the pertinent Green's function $G(\mathbf{r}, \mathbf{r}')$ has to satisfy the condition $G(\mathbf{r}, \mathbf{r}')|_{z=0}=0$, in contrast to the "geoelectrical condition" $\partial g(\mathbf{r}, \mathbf{r}')/\partial z|_{z=0}=0$. For the case of a halfspace, we have

$$G(\mathbf{r}, \mathbf{r}') = R^{-1} - R_+^{-1} \quad \text{(3D body)}, \tag{31}$$

$$\bar{G}(P, Q) = \tfrac{1}{2} \ln \left[\frac{(x-x')^2+(z+z')^2}{(x-x')^2+(z-z')^2}\right] \quad \text{(2D body)}. \tag{32}$$

The computer program which was developed for the geoelectrical problem, can be easily adopted to the geothermal one.

Numerical calculation

We have proved the applicability of the method presented here for some cases of block-like disturbing bodies, embedded in a conductive halfspace or in a two-layered Earth. The primary electric field was considered to be uniform, x-polarized, so that its potential is:

$$V(\mathbf{r}) = -E_0 \cdot x. \tag{33}$$

Such an electric field approximates the long-period MT field with large penetration depth (in comparison to the dimensions of the block). The method of computation and results are given in Hvoždara (1983a). The results

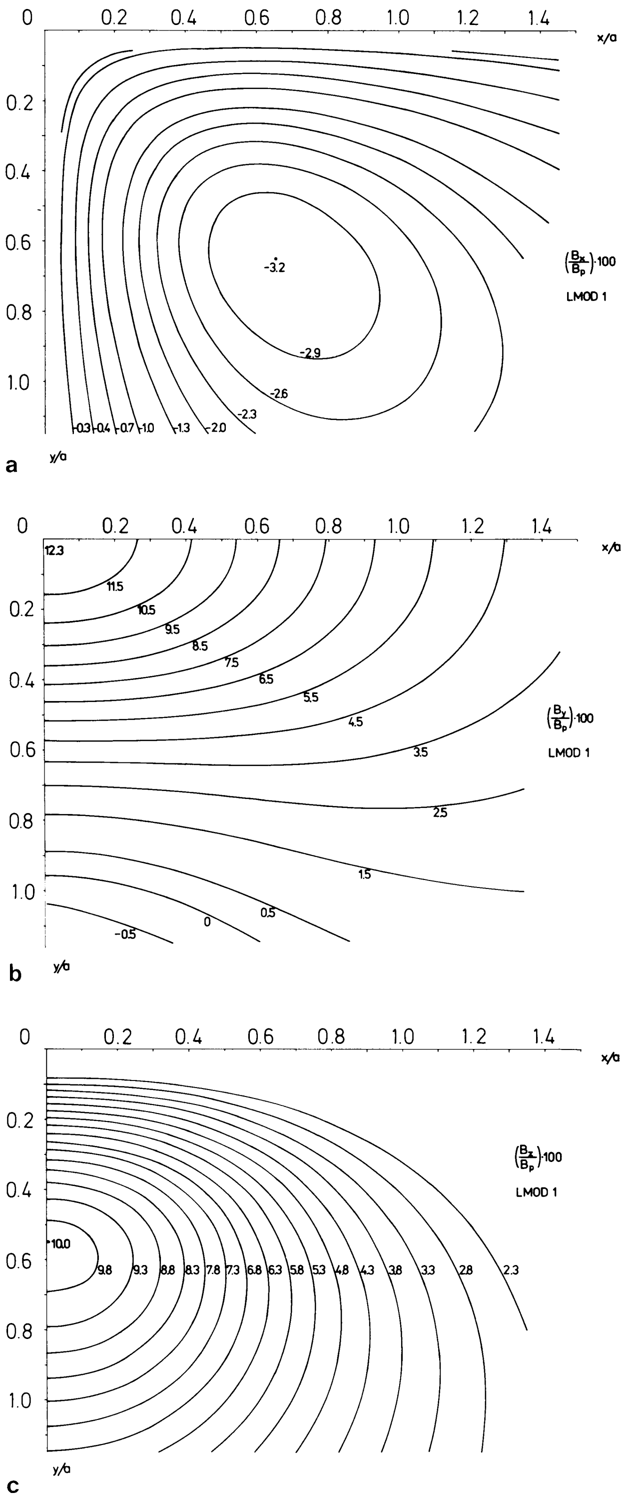

Fig. 4a–c. Isolines of B_x/B_p, B_y/B_p, B_z/B_p (multiplied by 100) on the surface $z=0$ in the quadrant $x \geqq 0$, $y \geqq 0$. In other quadrants B_x changes its sign like sign $(x \cdot y)$, B_y is symmetrical, B_z changes its sign like sign (y)

show that, in the halfspace with an embedded block, the potential (33) induces an anomalous field which is given by the sum of the field of an x-oriented electric dipole located near the centre of the block and the field of the mirror image of the "block dipole" due to plane $z=0$; i.e. its potential is:

$$U^*(\mathbf{r}) \approx p_x \cdot (4\pi\sigma_1)^{-1} \cdot [R^{-3} + R_+^{-3}] \cdot x, \qquad (34)$$

where $R^2 = x^2 + y^2 + (z - h_T)^2$, $R_+^2 = x^2 + y^2 + (z + h_T)^2$ and h_T is the depth of the block centre. The asymptotic value for the dipole moment p_x was given in the form:

$$p_x \approx 3\,v_T(\sigma_T/\sigma_1 - 1) \cdot (2 + \sigma_T/\sigma_1)^{-1} \cdot \sigma_1 E_0, \qquad (35)$$

where $v_T = a \cdot b \cdot c$ is the volume of the block. This dipole moment can be used in the approximate calculation of the anomalous magnetic field on the surface $z=0$, by means of Eq. (27a):

$$\begin{aligned} H_x^*(\mathbf{r}) &\approx p_x/(4\pi) \cdot h_{xx}(\mathbf{r}, \mathbf{r}'), \\ H_y^*(\mathbf{r}) &\approx p_x/(4\pi) \cdot h_{yx}(\mathbf{r}, \mathbf{r}'), \qquad (36) \\ H_z^*(\mathbf{r}) &\approx p_x/(4\pi) \cdot h_{zx}(\mathbf{r}, \mathbf{r}'), \end{aligned}$$

putting the coordinates of the source point $Q(\mathbf{r}') = (0, 0, h_T)$.

For comparison of the time-harmonic field and its stationary approximation, we present curves of the ratio $|E_x/E_0|$ for profile $y=0$ at the surface of the halfspace in the case of a model of a well-conducting block in a low-conductive halfspace, proposed by Stodt et al. (1981); see Fig. 1. We have calculated the curves for periods $T=0.1$, 1.0 and 10 s by the vector integral method, as in Hvoždara (1981a, b). The curves are close to the curve for the stationary approximation ($T=\infty$) calculated by the scalar potential method. Similar closeness was recognized in spatial distribution of other components of the EM field as well. It is worthwhile to note that the computation by the scalar potential method is about five times faster in comparison with the vector integral method.

The calculation for the case of a two-layered Earth is slightly more complicated because of more terms in Green's function, Eq. (18). The block was first considered to be fully embedded in the substratum $z>h$, and then in contact with the upper layer as shown in Fig. 2. This is the model of a block-like depression of the layer into the substratum. For this "contact case" we must be careful in the application of basic integral equations, Eqs. (19) and (23). They are applicable only to the part $S_2 \subset S$, which is not in contact with the planar boundary $z=h$. Using the limit $P(\mathbf{r}) \to S_{1-}$ in Eq. (14) for the contact part $S_1 \subset S$ ($S = S_1 \cup S_2$), we obtain the following integral equation:

$$U_T(\mathbf{r}) = a_2 V_2(\mathbf{r}) + b_2/(2\pi) \cdot \oint_S U_T(\mathbf{r}')\, \partial g_2(\mathbf{r}, \mathbf{r}')/\partial n'\, dS', \qquad (37)$$

where $a_2 = (\sigma_1 + \sigma_2)/(\sigma_1 + \sigma_T)$, and $b_2 = (1 - \sigma_T/\sigma_2) \cdot a_2/2 = \beta/(1 - q\beta)$. Here we have taken into account another singular term $q[(x-x')^2 + (y-y')^2 + (z+z'-2h)^2]^{-1/2}$ in $g_2(\mathbf{r}, \mathbf{r}')$ at $z'=h$ and $z \to z'$ apart from term R^{-1}; see Eq. (18). Using the auxiliary function $f(\mathbf{r})$, defined by Eq. (20), we obtain, for $P(\mathbf{r}) \in S_2$, the same equation as Eq. (23), but for $P \in \mathbf{S}_1$ we will have the integral equation with different coefficients:

$$\begin{aligned} f(\mathbf{r}) = {} & 2 b_2 [V_2(\mathbf{r}) - q_0] + b_2/(2\pi) \\ & \cdot \oint_S f(\mathbf{r}')\, \partial g_2(\mathbf{r}, \mathbf{r}')/\partial n'\, dS', \qquad P(\mathbf{r}) \in S_1. \qquad (38) \end{aligned}$$

For the depression model we put $\sigma_T = \sigma_1$. Its pa-

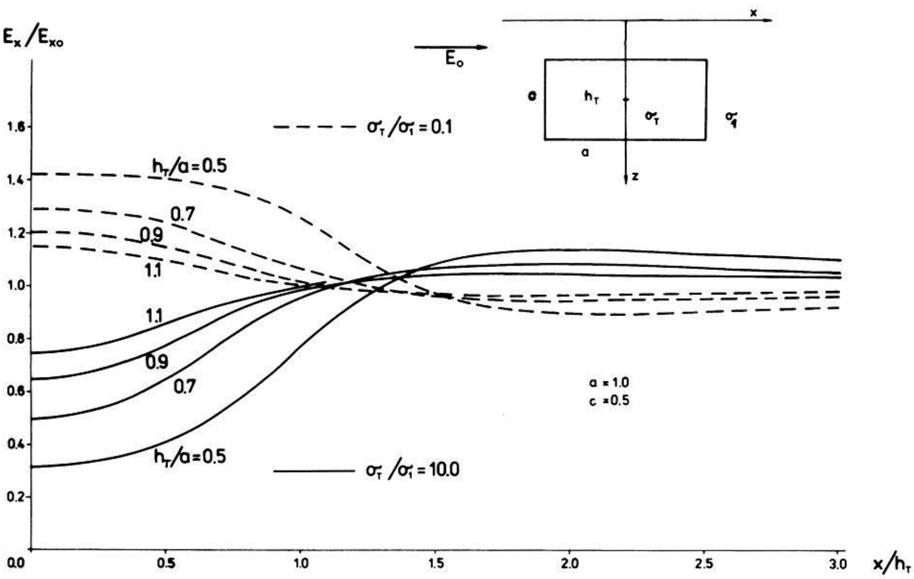

Fig. 5. Model of a 2D block in the halfspace with uniform inducing telluric field and corresponding curves E_x/E_{x0} at $z=0$; for various depths and for well-conducting *(solid lines)* or low-conducting *(dashed lines)* blocks

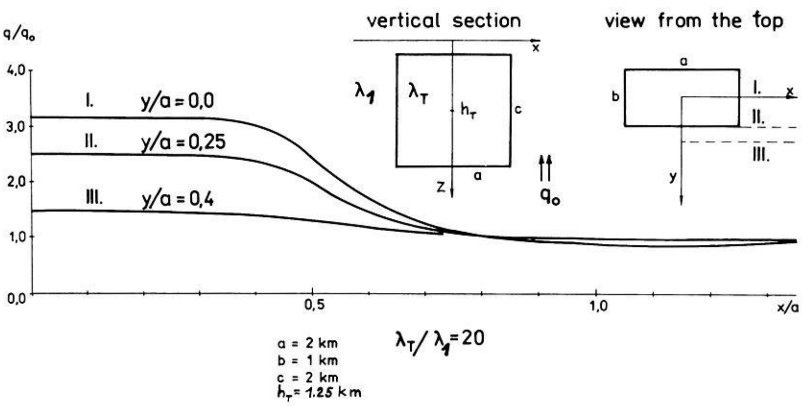

Fig. 6. Surface heat flow profiles for 3D block of the same geometrical parameters as in Fig. 1 and $\lambda_T/\lambda_1=20$

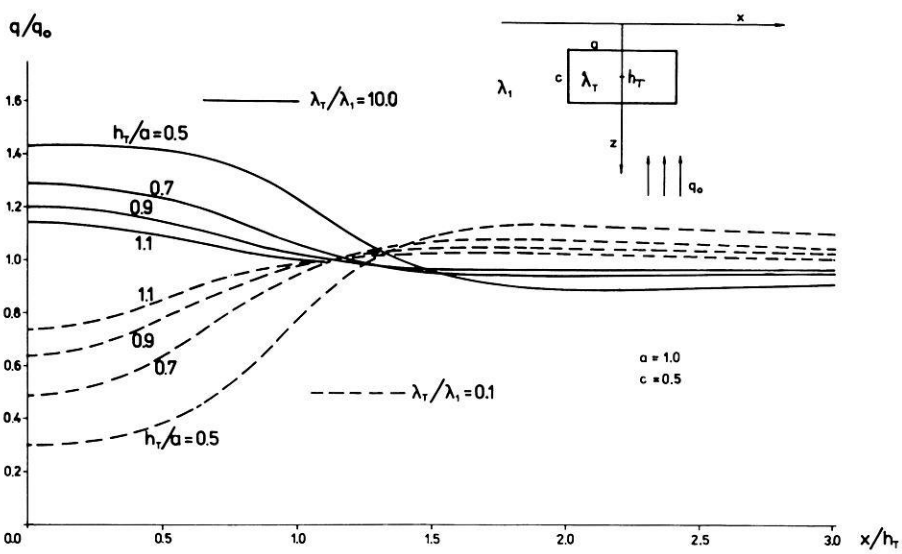

Fig. 7. Surface heat flow profiles for 2D blocks of the same geometrical parameters as in Fig. 5; $\lambda_T/\lambda_1=10$ – *solid lines*, $\lambda_T/\lambda_1=0.1$ – *dashed lines*

rameters are given in Fig. 2, the side a of the block being chosen as a unit of length. Isolines of the x- and y-components of the $\mathbf{E}=\mathbf{E}_0+\mathbf{E}^*$ are given in Fig. 3a and b. We can see that the spatial distribution of the components corresponds to $-\mathrm{grad}\,[V(\mathbf{r})+U^*(\mathbf{r})]$, where U^* is given by Eq. (34); the depression has a pronounced effect on the telluric field. The corresponding anomalous magnetic field $\mathbf{B}=\mu_0\cdot\mathbf{H}^*$ at the surface was normalized with respect to the magnetic field $B_p=\mu_0\sigma_1 E_{x0}\cdot a$, and isolines for x-, y- and z-components are given in Fig. 4a–c. Their spatial distribution nearly corresponds to Eq. (36) – to the x-oriented current dipole located in the centre of the block.

As an example of computation for 2D problems,

Fig. 5 illustrates graphs for the intensity of the electric field due to potential (33) in a conductive half-space with a 2D block. The central axis of the block – a well- or low-conducting block – is considered at four depths: $h_T/a = 0.5$, 0.7, 0.9 and 1.1. The results were obtained by Eqs. (28)–(30) using Green's function for the halfspace:

$$\bar{g}(P, Q) = \ln[(x-x')^2 + (z-z')^2]^{-1/2} + \ln[(x-x')^2 + (z+z')^2]^{-1/2}.$$

The anomaly in the electric field is pronounced, but the anomalous magnetic field in this case of 2D inhomogeneity is zero at the surface. This is in agreement with the H-polarization case in 2D induction problems.

Using the mathematical similarity with stationary geothermal problems, the heat flow anomalies due to 3D and 2D blocks of heat conductivity λ_T embedded in a halfspace of heat conductivity λ_1 were calculated. The undisturbed temperature distribution in the halfspace was assumed to be of the form $V(\mathbf{r}) = (q_0/\lambda_1) \cdot z$, q_0 now being the undisturbed heat flow density. For the 3D case, a block of the same geometrical parameters as for Fig. 1 was used and $\lambda_T/\lambda_1 = 20$. The disturbed temperature field $U(\mathbf{r})$ was calculated by Eqs. (21) and (23) using Eq. (31) as Green's function $G(\mathbf{r}, \mathbf{r}')$, which provides zero value of $U(\mathbf{r})$ at $z = 0$. The surface heat flow q is given as $q = \lambda_1 [\partial U/\partial z]_{z=0}$. Results of the numerical calculations are given for three profiles in Fig. 6.

Perturbation of the temperature field due to the 2D block was calculated by Eqs. (28)–(30) using Eq. (32) as the pertinent Green's function. The geometrical parameters of the block are the same as in Fig. 5 and $\lambda_T/\lambda_1 = 10$, or 0.1. The profile curves of q/q_0 for various depths of the block axis are plotted in Fig. 7.

From the theory of solids it is known that the heat conductivity λ is proportional to the electrical conductivity σ for semiconductors and metals. So we can state that at least some part of the heat flow anomaly, which is usually connected with the observed electrical conductivity anomaly, could be explained in this manner.

In conclusion, we can confirm that the double-layer potential method is a useful tool for the effective solution of many geophysical potential problems.

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