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Finite parallel conductivity in the open magnetosphere

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Abstract. A pattern of convection is calculated for a magnetospheric model with open magnetic field lines going from the polar cap to the solar wind. A uniform anti-sunward flow at the open field lines is assumed as a source of convection. A layer with finite parallel conductivity is situated below the source. The ionospheric convection has a two-vortex structure, but the convective flow across the polar cap appears to be smaller than the source flow due to a rise of parallel electric fields in the field-aligned electric currents at the polar cap boundary. The return convective flow in the region of closed magnetic field lines is also smaller than the source flow. To conserve the flow continuity two narrow (with a width of about 10 km projected onto the ionosphere) intense return convective streams arise above the finite parallel conductivity layer at the boundary of open and closed magnetic field lines. The transverse potential difference in the streams is equal to the parallel potential difference in the field-aligned currents at the polar cap boundary. Another consequence of the finite parallel conductivity is a shift of centres of the ionospheric convection vortices into the polar cap.

Key words: Magnetosphere – Ionosphere – Electric field – Field-aligned current – Parallel conductivity – Convection

Introduction

Dungey (1961) supposed that reconnection of the Earth's and interplanetary magnetic fields in the framework of an open magnetosphere model gives rise to magnetospheric plasma convection. Intensification of the convection during periods of southward IMF, when the reconnection is most efficient, was confirmed experimentally (Heppner, 1972). One of the consequences of convection is field-aligned currents at the polar cap boundary. Iijima and Potemra (1976) named them the currents of Region 1. Usually these currents are theoretically studied under conditions of infinite conductivity parallel to the magnetic field. Meanwhile, parallel conductivity is often reduced so that a potential difference of up to 10 kV arises along high-latitude magnetic field lines (Mozer et al., 1980). Possible mechanisms of generation of electric fields $E_{\parallel}$ were reviewed by Block and Fälthammar (1976). The role of $E_{\parallel}$ in large-scale convection and current was studied in a few papers. Lyons (1980) has assumed a potential distribution with $\operatorname{div}\mathbf{E} \neq 0$ at some distance from the ionosphere and has obtained a broadening of the sheet of field-aligned currents under the action of $E_{\parallel}$. These calculations may be applied to Region 1 currents. Chiu et al. (1981) solved a similar problem but their task was to construct an auroral arc model. Maltsev (1985) has shown that $E_{\parallel}$ in Region 2 currents produces two isolated convective vortices in the inner magnetosphere as well as narrow intense azimuthal convective streams along both sides of the sheet of Region 2 currents.

The aim of this paper is to investigate the effect of $E_{\parallel}$ on convection near Region 1 of field-aligned currents. In this respect, our problem is similar to the one studied by Lyons (1980) but boundary conditions are different. Contrary to Lyons (1980) we assume that the sources of convection are localized in the area of open magnetic field lines. The parallel electric field will be taken into account by introducing a layer with finite parallel conductivity, the specific mechanism of which is of no importance to us.

Model and basic equations

A layer of thickness b with finite parallel conductivity $\sigma_{\parallel}$ is situated not very far from the ionosphere. Above and below the layer the parallel conductivity is infinitely large. The area where open magnetic field lines cross the ionosphere is a circle of radius r_0. Above the layer with finite parallel conductivity the following potential distribution is assumed:

$$\varphi_m = E_0 r \sin\lambda \quad \text{for } r < r_0, \tag{1}$$

where r and E_0 are the distance and the electric field, respectively, projected onto the ionosphere, $r=0$ at the pole, λ is longitude ($\lambda=0$ at midnight). The difference between the ionospheric φ_i and magnetospheric φ_m potentials is of the form

$$\varphi_i - \varphi_m = \frac{b}{\sigma_{\parallel}} j_{\parallel}, \tag{2}$$

where $j_{\parallel}$ is the density of field-aligned currents. The ratio $\sigma_{\parallel}/b$ is considered to be independent of the horizontal coordinates.

For the ionospheric potential we have the equation

$$\operatorname{div} \hat{\Sigma} \nabla \varphi_i = j_{\|}, \tag{3}$$

where $\hat{\Sigma}$ is the tensor of height-integrated ionospheric conductivity. For simplicity we assume that the ionosphere is a uniformly conducting plane; the magnetic field is perpendicular to the ionosphere. Hence Eq. (3) is rewritten as

$$\Sigma_P \Delta_{\perp} \varphi_i = j_{\|}, \tag{4}$$

where Σ_P is the Pedersen conductivity, $\Delta_{\perp}$ is the two-dimensional Laplace operator.

We are required to find the magnetospheric potential φ_m in the closed magnetic field line area $(r > r_0)$ as well as the ionospheric potential φ_i. Equations (2) and (4) are insufficient for solving the problem. Just one more relationship between $j_{\|}$, φ_m and φ_i in the region $r > r_0$ is necessary. We shall consider two variants of the relationship. In the next section the simplest case when there are no current sources in the closed field line area is studied, hence $j_{\|} = 0$ for $r > r_0$. In the subsequent section, inertia currents are taken into account.

Calculation of the potential

Equations (2) and (4) yield

$$\Delta_{\perp} \varphi_i = k^2 (\varphi_i - \varphi_m), \tag{5}$$

where

$$k = \left(\frac{\sigma_{\|}}{\Sigma_P b} \right)^{1/2}. \tag{6}$$

Similar expressions were obtained by Chiu et al. (1981). In this section we neglect currents in the area of closed field lines. As a result, the equipotentially of these lines takes place:

$$\varphi_m = \varphi_i \quad \text{for } r > r_0. \tag{7}$$

Substitution of φ_m from Eqs. (1) and (7) into Eq. (5), as well as the continuity of φ_i and $\partial \varphi_i / \partial r$, yield the ionospheric potential:

$$\begin{aligned} \varphi_i &= E_0 \left[r - \frac{2}{k} \frac{I_1(kr)}{I_0(kr_0)} \right] \sin \lambda \quad \text{for } r \leqq r_0, \\ \varphi_i &= E_0 \frac{I_2(kr_0)}{I_0(kr_0)} \frac{r_0^2}{r} \sin \lambda \quad \text{for } r \geqq r_0, \end{aligned} \tag{8}$$

where I is the modified Bessel function.

From Eqs. (1), (2), (7) and (8) we get the field-aligned current

$$\begin{aligned} j_{\|} &= -2 \Sigma_p E_0 k \frac{I_1(kr)}{I_0(kr_0)} \sin \lambda \quad \text{for } r < r_0, \\ j_{\|} &= 0 \quad \text{for } r > r_0. \end{aligned} \tag{9}$$

Substitution of Eqs. (9) and (2) into Eq. (6) yields the relation, valid for $k r_0 \gg 1$,

$$k \approx \frac{2 E_0}{\varphi_{\| \max}}, \tag{10}$$

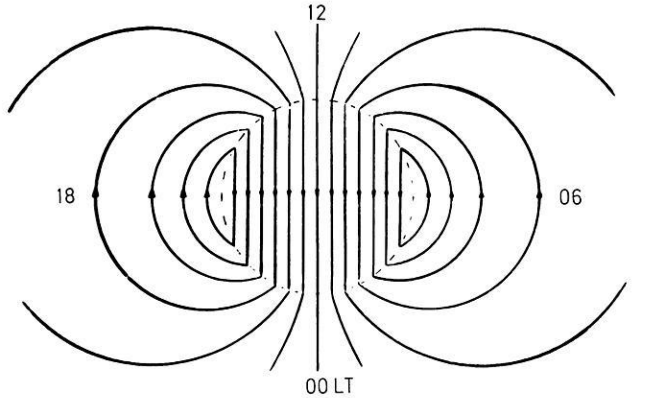

Fig. 1. The potential distribution in the ionosphere and magnetosphere in the case of infinitely large parallel conductivity. *Arrows* indicate the direction of the convective flow

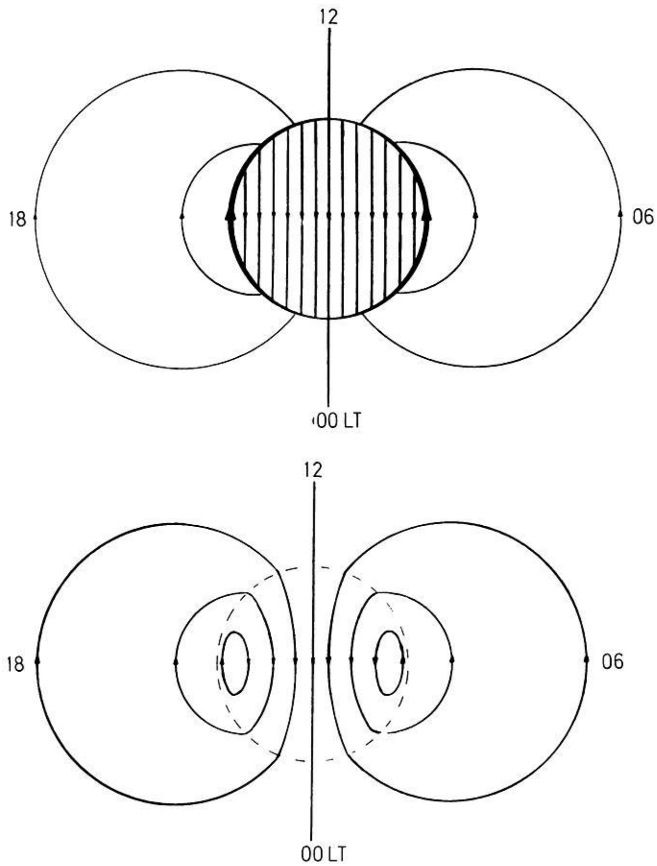

Fig. 2a, b. The potential distribution in the case of finite parallel conductivity: **a** the potential in the magnetosphere; **b** the potential in the ionosphere

where $\varphi_{\| \max}$ is the maximum magnitude of the parallel potential difference. Note that $\varphi_i - \varphi_m = \mp \varphi\|_{\max}$ at the points $r = r_0$, $\lambda = \pm \pi/2$, i.e. at the dawn and dusk boundaries of open magnetic field lines, respectively.

In Fig. 1 the potential distribution is shown for the case $\sigma_{\|} = \infty$. Potentials in the ionosphere and in the magnetosphere evidently coincide. The electric field is uniform inside the area of open magnetic field lines and resembles a field of a two-dimensional dipole outside this area.

In Fig. 2 the potential is shown for the case $kr_0=3$. It is interesting that the magnetospheric potential (Fig. 2a) has a discontinuity at the boundary of open and closed magnetic field lines. The magnitude of the discontinuity, taking Eq. (7) into account, is

$$\delta\varphi_m=\varphi_m|_{r=r_0-0}-\varphi_m|_{r=r_0+0}=(\varphi_m-\varphi_i)|_{r=r_0-0}, \tag{11}$$

i.e. it is equal to the parallel potential difference near the boundary of open field lines. A strong spike in the electric field $\delta E_\perp=\varphi_\parallel/\delta r$ occurs at the boundary. The characteristic scale δr is determined, in particular, by inertia of ions. Intense sunward convective streams are connected with the electric field spike.

The corresponding ionospheric potential is shown in Fig. 2b. The centres of the convective vortices appear to be shifted poleward. It is not difficult to show that the centres are situated at a distance (providing $kr_0 \gg 1$)

$$r\approx r_0-\frac{\ln 2}{k}.$$

Influence of inertia currents

In the previous section, field-aligned currents in the area of closed magnetic field lines were neglected. As a result, an infinitely large spike of the magnetospheric electric field arose at the boundary of open and closed field lines. As seen from Fig. 2a, the convection lines cross the boundary. Plasma passing across the region of a strong electric field undergoes strong accelerations which give rise to inertia currents. The inertia currents, in their turn, produce additional small-scale field-aligned currents. Let us consider small-scale currents at the night side where the plasma drifts from the polar cap to the closed field line area.

The inertia current is determined by the well-known expression

$$\mathbf{j}_\perp=\frac{\rho c^2}{B^2}\frac{d\mathbf{E}}{dt}, \tag{12}$$

where ρ is the plasma density, B is the ambient magnetic field. The process is assumed to be stationary ($\partial/\partial t=0$). The characteristic time of electric field variations in the coordinate system of drifting plasma is assumed to be large compared with the Alfvén resonance period, $T=4l/V_A$, where l is the half-length of the magnetic field line and $V_A=B/\sqrt{4\pi\rho}$ is the Alfvén velocity. In this case we may neglect deformations of the magnetic field lines caused by the currents.

Integration of Eq. (12) along the magnetic field line from the finite parallel conductivity layer to the equatorial plane yields the total current transverse to a magnetic flux tube of unit ionospheric area

$$\mathbf{J}_\perp=\frac{c^2}{4\pi}(\mathbf{v}\nabla)\,\mathbf{E}_m\int_0^l\frac{dz}{V_A^2}. \tag{13}$$

The continuity condition gives the field-aligned currents at the ionosphere level to be

$$j_\parallel=\operatorname{div}\mathbf{J}_\perp=-\frac{c^2}{4\pi}(\mathbf{v}\nabla)\,\Delta_\perp\varphi_m\int_0^l\frac{dz}{V_A^2}. \tag{14}$$

Combining Eqs. (2), (4) and (14), we get

$$\frac{c^2}{4\pi}\left(\int_0^l\frac{dz}{V_A^2}\right)(\mathbf{v}\nabla)\left(\frac{b}{\sigma_\parallel}\Delta_\perp j_\parallel-\frac{1}{\Sigma_p}j_\parallel\right)-j_\parallel=0. \tag{15}$$

Let us suppose that the characteristic scale of changes of the current $j_\parallel$ along r is much smaller than r. In this case, Eq. (15) is rewritten as

$$\Lambda\left(\frac{1}{k^2}\frac{d^3j_\parallel}{dr^3}-\frac{dj_\parallel}{dr}\right)-j_\parallel=0, \tag{16}$$

where

$$\Lambda=\frac{c^2}{4\pi}\frac{v_r}{\Sigma_p}\int_0^l\frac{dz}{V_A^2}. \tag{17}$$

Equation (16) will be solved under the following boundary conditions:

$$j_\parallel|_{r=r_0+0}=j_\parallel|_{r=r_0-0}, \tag{18}$$

$$j_\parallel|_{r\to\infty}=0, \tag{19}$$

$$\int_{r_0}^{\infty}j_\parallel\,dr=0. \tag{20}$$

Condition (18) is equivalent to the condition of absence of infinitely large spikes of the transverse electric field. Condition (20) means that the current in the magnetosphere does not flow from the open field line area into the closed field line area.

Let us assume $\Sigma_p=1$ mho, $V_A=10^3$ km/s, $T=4l/V_A=100$ s. For midnight, where $v_r\approx 0.5$ km, we get $\Lambda\approx 1$ km. For dawn and dusk meridians, where $v_r\approx 0$, we get $\Lambda\approx 0$. We shall study disturbances with horizontal dimensions much larger than 1 km, hence the second term in Eq. (16) may be neglected when compared with the third term. The solution of Eq. (16), under conditions (18)–(20), is

$$j_\parallel(r>r_0)=j_\parallel(r_0)\frac{2}{\sqrt{3}}\cdot e^{-(\alpha/2)(r-r_0)}\cos\left[\frac{\alpha\sqrt{3}}{2}(r-r_0)+\frac{\pi}{6}\right], \tag{21}$$

where

$$\alpha=\left(\frac{k^2}{\Lambda}\right)^{1/3}. \tag{22}$$

From Eq. (2), taking Eqs. (21) and (10) into account, we get the magnetospheric electric field (providing $kr_0\gg 1$)

$$E_{mr}=E_{0r}\quad\text{for } r<r_0,$$

$$E_{mr}=-E_{0r}\left\{\frac{r_0^2}{r^2}+\left(\frac{4\varphi_{\parallel\,\max}}{E_0\Lambda}\right)^{1/3}\cdot\frac{2}{\sqrt{3}}e^{-(\alpha/2)(r-r_0)}\cos\left[\frac{\alpha\sqrt{3}}{2}(r-r_0)+\frac{\pi}{3}\right]\right\} \tag{23}$$

for $r>r_0$,

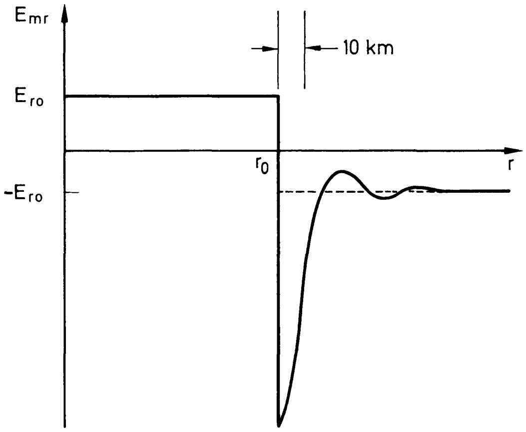

Fig. 3. The magnetospheric electric field distribution near the boundary of open and closed magnetic field lines

where $E_{0r} = -E_0 \sin\lambda$ is the radial component of the electric field in the polar cap. In Fig. 3 the $E_{mr}(r)$ dependence is shown. The following parameters are chosen: $\varphi_{\parallel\,\max} = 2\,\mathrm{kV}$, $E_0 = 30\,\mathrm{V/km}$, $\Lambda = 1\,\mathrm{km}$. In this case we have $k \approx 3 \times 10^{-2}\,\mathrm{km}^{-1}$, $\alpha \approx 10^{-1}\,\mathrm{km}^{-1}$. Quickly damping oscillations of the electric field arise near the boundary. The characteristic scale of oscillations is $\alpha^{-1} \approx 10\,\mathrm{km}$. The first spike is about six times the large-scale field. It should be noted that there are no small-scale spikes of the field in the ionosphere.

Oscillations shown in Fig. 3 are the result of superposition of Alfvén waves carried by the convection and undergoing repeated reflections from the conjugate hemispheres. The waves are damped due to ohmic losses in the ionosphere and in the finite parallel conductivity layer.

Discussion

In Fig. 4 the potential distribution in the model by Lyons (1980) (Fig. 4a) is compared with that in the present model (Fig. 4b). The solid line is the magnetospheric potential, the dashed line is the ionospheric one. The potential in Fig. 4a is continuous. The magnetospheric potential in Fig. 4b has a jump at the boundary of open and closed lines. The potential jump must be registered by satellites as a strong spike of the transverse electric field.

Spikes of the transverse electric field of the magnitude of several hundred millivolts per metre with a width of $\sim 10\,\mathrm{km}$ are often observed at altitudes of 2,000–8,000 km (Mozer et al., 1980). Their generation mechanism is not clear because fields of such magnitude are observed neither in the ionosphere nor in the solar wind. A possible cause of the spikes is an inhomogeneity of field-aligned current flowing through the layer with finite parallel conductivity. In the open magnetosphere model the field-aligned current is, most likely, sharply inhomogeneous and, consequently, is capable of generating spikes of $E_\perp$.

Chmyrev et al. (1983) reported observations of two electric field spikes of magnitude $\sim 250\,\mathrm{mV/m}$ with a width of 1–10 km at altitudes of 800–900 km. The electric spikes were accompanied by magnetic ones with an

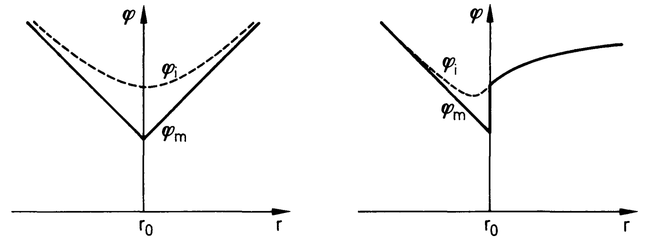

Fig. 4a, b. Distribution of the magnetospheric (*solid lines*) and ionospheric (*dashed lines*) potential in different models: **a** the model of Lyons (1980); **b** our model

amplitude of $\sim 300\,\mathrm{nT}$. Three analogous spikes of magnitudes of 200–500 nT and with a width of 2–8 km were observed at the same altitudes according to Volokitin et al. (1984). In all cases the magnetic disturbances were transverse to the ambient magnetic field, hence they were caused by field-aligned currents. It is not difficult to show that the field-aligned currents, Eq. (14), give the magnetic disturbance

$$B_\lambda = -\frac{4\pi \Sigma_p \Lambda}{c} \frac{\partial E_{mr}}{\partial r}. \tag{24}$$

Assuming $\Sigma_p = 1\,\mathrm{mho}$, $\Lambda = 1\,\mathrm{km}$, $\alpha = 0.3\,\mathrm{km}^{-1}$, $E_{mr} = 200\,\mathrm{mV/m}$ we get magnetic oscillations with amplitude $B_\lambda \approx 100\,\mathrm{nT}$. It should be mentioned that the magnetic spikes in the given model must be observed both above and below the finite parallel conductivity layer, whereas the electric spikes only above it.

Conclusions

A decrease of the parallel conductivity in an open magnetosphere model gives rise to the following consequences:

1) The centres of convective vortices are not located at the boundary of the open field line area. They are shifted into this area.

2) Narrow intense convective sunward streams arise above the finite parallel conductivity layer at the dawn and dusk boundaries of the closed field line area. The characteristic width of the streams is about 10 km. The electric field in the streams is several hundred millivolts per metre. Small-scale ($\sim 10\,\mathrm{km}$) variations of field-aligned currents and of transverse magnetic disturbances with magnitude of $\sim 100\,\mathrm{nT}$ are connected with the streams.

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