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Inverse dynamic problems and seismic methods for determination of the structure of a medium

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Abstract. In this work we discuss the problems of applying dynamic inversion methods in seismic exploration. The linearized inversion methods, rapidly developed in recent years, are still of very limited practical use. This is associated particularly with the presence, in real data, of powerful regular-noise background which is neglected in linearized solutions. At the same time, the usual treatment by the CDP-migration does not provide the required quality in images of seismic sections in complicated situations, particularly in the case of a 3-D strongly inhomogeneous structure of a medium.

The proposed method of imaging consists in reducing the inverse dynamic problem in its classical statement to a problem of reconstructing the stable functionals of the velocity structure of a medium, i.e. the dynamic images of its inhomogeneities. The relative redundancy of multifold seismic observation systems is used for dynamic filtration of regular noise. If a medium is complex-structured, the physical parameters of the equation of the useful part of the field are corrected in the process of treatment. The imaging transformations are supplemented here with specific continuation of the field in the inhomogeneous medium. The practical treatment is interactive. In connection with 3-D inverse dynamic imaging it is sufficient to observe the field, using an areal system of parallel profiles of multifold coverage.

Key words: Seismic image inversion – Double wave migration – Weighted spatial-frequency stacking

Introduction

At the present time, common-depth-point (CDP) and wave migration methods are most widely used for processing and interpretation of wave fields. However, recently in seismic exploration problems have arisen that extend beyond the limits of these seismic data processing methods. These are problems of seismic stratigraphy, problems of searching for geological structures in complicated spatial-configuration regions, etc. Kinematic simplification of schemes defining useful waves, representation of an inhomogeneous medium by average-velocity models and other assumptions used in the CDP-migration methods may, more often than not, substantially distort results of processing.

The most complete and exact information on physical characteristics of inhomogeneous media can be provided by solving inverse dynamic problems in general formulation. Here we mean problems of defining variable coefficients (physical parameters of a medium) in differential equations according to wavefields described by these equations. Theory and methods of numerical solution of such problems developed up till now, and results obtained, allow us to firmly restore the structure of vertically inhomogeneous media (Alekseev, 1967; Alekseev and Dobrinsky, 1975). An important issue which remains to be considered is the development of methods for multidimensional inverse dynamic problems and the applicability of corresponding transformations in real data processing. A serious handicap, besides the mathematical difficulties, remains in the rigid requirements of acceptable models of 2-D and 3-D media to details of observations and accuracy of dynamic wavefield recordings.

The theory of solving inverse dynamic problems admits the presence of irregular noise in the initial data. It is assumed that minimization of discrepancies between an observed and calculated field (using an iterative method), in the case of a unique solution of the inverse problem, is a sufficient condition to find an acceptable solution (Aki and Richards, 1980). However, if the equations used are inadequate for real fields the solutions thus obtained can intrinsically differ from real ones, even if there is negligible irregular noise. This is because part of the recorded waves turns into the category of regular noise, not taken into account in the solution, which is the cause of incorrectness of the inversion. An example of incorrectness is the use of exact algorithms of acoustic inversion for processing an elastic wavefield. Obviously, minimization of discrepancies between values of the simulated acoustic field and the recorded elastic wavefield is not sufficient for the acoustic solutions to converge to some approximate values of the parameters of an elastic medium. Realistic seismic fields can be appreciably affected by imperfect elasticity, porosity, anisotropy and other physical properties of rocks. Disregarding the wave phenomena involved can also be the cause of incorrectness of exact

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solutions of inverse problems for the Lamé equations describing the seismic fields.

Approaches to the solution of dynamic problems in seismic exploration

We will consider schematically how a problem of wave-noise filtration in seismic exploration is solved from the point of view of inverse problems.

Let the observed field $U = Lv$ be described as the sum

$$U = U_0 + U_1 = L_0 v_0 + L_1 v_1,$$

where L, L_0 are, respectively, operators of a full (realistic) and particular (simulated) solution of the forward dynamic problem; v and v_0 are physical parameters of the realistic and model medium, respectively; $U_1 = L_1 v_1$ is the part of the observed field considered to be noise. We assume that an inverse operator L_0^{-1} of the simulated problem is constructed which is stable with respect to irregular noise and such that $L_0^{-1} U_0 = v_0$. Applying it to the field $U = U_0 + U_1$ will yield a distorted solution if, in the domain of definition of v_0, the values $\|L_0^{-1} U_1\| = \|L_0^{-1} L_1 v_1\|$ (i.e. the result of its influence on the field of regular noise) are comparable to or exceed the values of $\|v_0\|$ in some norm. Assume that there exist filtering operators M reducing the recorded field U to $MU = U_m + \varepsilon_m$ so that $U_m \approx L_m v_0$, where L_m is the operator of some special solution of the forward problem (in the general case, L_m can be different from L_0). To filter out the regular noise-field and find v_0, it is necessary to find operators M and L_m^{-1} such that $L_m^{-1} MU \approx v_0 + L_m^{-1} \varepsilon_m$ and, in the chosen norm, $\|v_0\| \gg \|L_m^{-1} \varepsilon_m\|$. Typical of this statement is that under special filtration of the observed field the inverse problem can be posed for an equation that is more easily handled than the initial (model) one (Alekseev, 1967). When part of the regular noise-field has characteristics which differ only slightly from those of the useful field, full filtration of noise may prove impracticable. In this case transformation of M and L_m^{-1} will lead to reconstruction of some functionals $c(v)$ of the realistic physical parameters of a medium. Though they do not imply an exact solution to the inverse problem the information about the medium carried by them may be of great importance in seismic prospecting.

An example of an extremely simple implementation of this approach can be CDP-migration processing. When seismograms are processed by the CDP method one usually classifies multiple, surface and, sometimes, converted waves as noise. CDP transforms, based on simple kinematic schemes, are used to process a full dynamic wavefield, i.e. the method is not purely kinematic. Filtration of the regular noise-field by stacking of seismograms along CDP time-distance curves is made possible due to redundancy of multifold observation systems (standard seismic exploration systems). For example, the 2D CDP section $U_m(l, t) = M_{\mathrm{CDP}} U(x_0, x, t)$ is formed from the field $U(x_0, x, t)$ (recorded on a profile line), dependent on three variables (here M_{CDP} is a CDP-stacking operator). The redundancy is used to stack the signal component of the field and to suppress the part of irregular wave noise whose kinematics is different from that of useful waves. However, the kinematic simplification of the CDP scheme often gives rise to a dynamic distortion of useful reflections on CDP sections. Besides, the size of the common reflection element is fairly small only for an insignificant dip of the reflecting boundaries. With the increase of the dip of the boundaries, dynamic distortion intensifies and the locality property of CDP sections is lost.

In further treatment of seismic data, the CDP sections are assumed to be a filtered field of primary reflected waves (to be recorded at points of wave excitation) against remaining noise background. In migrating the CDP sections they are taken to be the result of radiation of secondary wave sources distributed along reflecting boundaries. The inverse problem of migration usually reduces to determining values of the field at the moment $t = 0$ when these secondary radiations are "turned on". This allows one to obtain an image of the inhomogeneities of the medium, i.e. to image it. As shown by Alekseev and Tsybul'chik (1978), there is a certain connection between problems of imaging the structure of a medium and inverse problems of wave theory. So far an obvious correspondence between them has been found only in the case of simple models of a medium. Nevertheless, from our experience with using the CDP-migration methods it may be presumed that constructive approaches being employed in seismic exploration can be helpful in developing approximate methods for inverse dynamic problems.

By means of CDP-migration methods the inverse problem to determine elastic parameters of an inhomogeneous medium reduces, in fact, to simple problems of multidimensional filtration (stacking) of the field and to the subsequent reconstruction of special functionals of its structure (seismic sections). Methods of calculation of "velocity spectra" make it possible to determine also the mean (effective) velocity of wave propagation in a layered medium. Moreover, correlation between peculiarities of the pattern of stacking sections and the geology of sedimentary rocks is observed. Therefore, the formulation and solution of a more general problem, an inverse dynamic problem of reconstruction of multidimensional functional-images of complexly structured media, on the basis of actual wavefields of redundant observation systems is of theoretical and practical interest.

The imaging dynamic inversion

We will describe the propagation of waves in a spatially inhomogeneous medium by the equation

$$\Delta U - \frac{1}{v^2(\mathbf{R})} U_{tt} = -\delta(\mathbf{R} - \mathbf{R}_0) f(t); \qquad \mathbf{R} = (x, y, z), \quad \mathbf{R}_0 = (x_0, y_0, z_0), \tag{1}$$

where $v(\mathbf{R})$ is the wave propagation velocity, $f(t)$ is the form of the wave radiated by the wave source [$f(t) \equiv 0$ for $t < 0$] which may be at any present point $\mathbf{R}_0$. Let the velocity $v(\mathbf{R})$ take on a constant value v_0 outside some inhomogeneous restricted domain D. Having performed a Fourier transform over t, we give the resulting Helmholtz equation in the equivalent integral form:

$$U(\mathbf{R},\mathbf{R}_0,k)=\Phi(kv_0)\frac{e^{-ik|\mathbf{R}-\mathbf{R}_0|}}{4\pi|\mathbf{R}-\mathbf{R}_0|} -\frac{k^2}{4\pi}\int_D \tilde{v}(\mathbf{R}_1)\,U(\mathbf{R}_1,\mathbf{R}_0,k)\frac{e^{-ik|\mathbf{R}-\mathbf{R}_1|}}{|\mathbf{R}-\mathbf{R}_1|}dR_1. \tag{2}$$

Here $\Phi(\omega)=F[f(t)]$ is the spectrum of the wave, $k=\omega/v_0$ is the wavenumber, $\tilde{v}(\mathbf{R})=1-v_0^2/v^2(\mathbf{R})$ is the function of relative velocity variations in a medium, $dR_1=dx_1\,dy_1\,dz_1$.

Let there be on the plane $z=0$ a recorded field, excited on $z_0=$const. by independent sources. The inhomogeneous domain D lies below the planes of wave excitation and recording, i.e. $\tilde{v}(\mathbf{R})=0$ for $z<0$, $z<z_0$. We will assume that the signal $f(t)$ in the sources and the value of the constant velocity v_0 near the observation system are known. The frequency spectrum of the recorded field

$$U(\mathbf{r},\mathbf{r}_0,k)=U(\mathbf{R},\mathbf{R}_0,k)|_{z_0=\text{const},\,z=0}$$
$$[\mathbf{r}=(x,y),\ \mathbf{r}_0=(x_0,y_0)]$$

is determined according to Eq. (2), which is looked upon as a non-classical integral equation concerning the velocity function $\tilde{v}(\mathbf{R})$. The unknown field in the medium, $U(\mathbf{R}_1,\mathbf{R}_0,k)$, which enters into the expression under the integral in Eq. (2), depends on the medium's velocity structure $v(\mathbf{R})$, which in turn must be determined from the field on the surface $z=0$. Hence, it is necessary to specify an assumption about the form of the field in the inhomogeneous medium. We will assume that in some part D_1 of the medium below sources and receivers it has a composition that is sufficiently similar to the downward field of sources in an effectively homogeneous volume, i.e.

$$U(\mathbf{R}_1,\mathbf{R}_0,k)\approx\Phi(kv_0)\frac{e^{-ik|\mathbf{R}_1-\mathbf{R}_0|}}{4\pi|\mathbf{R}_1-\mathbf{R}_0|}+U_1(\mathbf{R}_1,\mathbf{R}_0,k). \tag{3}$$

Within the framework of the problem of reconstruction of functional-images of the medium's structure, stated below, the condition of "sufficient similarity" is expressible by a Rayleigh criterion. According to this criterion, the system which forms the image is assumed to be fairly good if phase distortions of the field introduced by it are not greater than $\pi/4$. In our case, this means that the phase distortion of the downward field in D_1 due to inhomogeneity of the upper part of the medium must be less than $\pi/4$. From here there follow certain limitations on the degree of inhomogeneity and sizes of the domain D_1 (with respect to the wavelengths of the primary field). The composition of the velocity structure of the medium remains arbitrary enough to include both scattering elements and reflecting boundaries of any configuration.

In a series of papers (e.g. Alekseev, 1967; Zapreev, 1977; Raz, 1981; Clayton and Stolt, 1981) a low-frequency approximation (the condition of weak wave scattering inside a medium, called the Born-Rayleigh approximation) is used to solve inverse problems reducing to equations similar to Eq. (2). According to the weak scattering condition, the field of secondary waves in a medium [$U_1(\mathbf{R}_1,\mathbf{R}_0,k)$ in Eq. (3)] must be negligibly small compared to the downward source field

$$\mathscr{E}_0(\mathbf{R}_1,\mathbf{R}_0,k)$$
$$=\Phi(kv_0)\exp(-ik|\mathbf{R}_1-\mathbf{R}_0|)/(4\pi|\mathbf{R}_1-\mathbf{R}_0|).$$

This requirement is fulfilled only in the case of low frequencies or insignificant variations of velocity all over the medium, i.e. if $\omega\to 0$ or $\tilde{v}(\mathbf{P})\ll 1$, $\mathbf{R}\in D$ and if the dimensions of the whole inhomogeneity domain D are fairly small. Hence, the assumption about weak scattering substantially limits the form of the model of a medium allowed to solve respective inverse problems. Besides, as was shown by Zapreev (1977), if the conditions of the Born approximation are fulfilled and the noise background is present, we can stably determine not the function $\tilde{v}(\mathbf{R})$ itself, but only its low-frequency (smooth) part. Here we will not require that the secondary field U_1 be small in the medium as compared to the downward field of sources. This implies that relative velocity variations in the inhomogeneous domain D on the whole are not assumed to be small. The influence of the field U_1 in the proposed nonlinearized statement of the inverse problem will be looked upon as an effect of strong regular noise. On account of this, the solution of the problem of determining the structure of a medium must include a procedure for suppressing the wave noise in the observed field. The possibility of directional filtration of a regular noise-field is determined by the redundancy of the observation system considered (the recorded 5-D field and the 3-D structure of the medium). Relative redundant information in the data allows us to include, in the solution algorithms, procedures of dynamic stacking of useful waves (those described by adopted wave equations) and suppressing the remaining noise.

As shown by Zapreev and Cheverda (1981), part of the secondary field U_1 in a medium may be similar to the primary downward field. That is, the field U_1 in Eq. (3) may have a common (generally, significant) component of the form $\tilde{c}(\mathbf{R}_1)\mathscr{E}_0(\mathbf{R}_1,\mathbf{R}_0,k)$ which depends on unknown wave properties $\tilde{c}(\mathbf{R})$ of the inhomogeneous medium in the vicinity of any point $\mathbf{R}\in D_1$. Thus, as a result of the inversion, in the general case, we will determine not the velocity function $\tilde{v}(\mathbf{R})$ itself, but a functional of the form $c(\mathbf{R})=\tilde{v}(\mathbf{R})[1+\tilde{c}(\mathbf{R})]$ which depends on local properties of the medium. Because the values of the functional show precisely local variations of the velocity structure of the medium, it is defined as an image which reflects inhomogeneities of the medium. The physical sense of the function of secondary scattering $\tilde{c}(\mathbf{R})$ [hence the functional $c(\mathbf{R})$] can be defined more precisely by assuming a law of reradiation of the field in a medium, i.e. the physical model of the strong field $U_1(\mathbf{R}_1,\mathbf{R}_0,k)$. To do this, we may suppose that all elements in D_1 are rather smooth and sloping reflecting boundaries or they scatter the field according to a well-known law. However, such a definition considerably narrows the possibilities of using the corresponding inversion algorithms in practice.

After we substitute Eq. (3) into the integral of

Eq. (2), considering all the adopted requirements and assumptions, we obtain the integral equation with respect to the functional $c(\mathbf{R})$

$$U(\mathbf{r}, \mathbf{r}_0, k) = -\frac{k^2 \Phi(k v_0)}{(4\pi)^2} \int_{D_1} c(\mathbf{R}_1) \frac{e^{-ik|\mathbf{R}_1 - \mathbf{R}_0|}}{|\mathbf{R}_1 - \mathbf{R}_0|} \cdot \frac{e^{-ik|\mathbf{r}-\mathbf{R}_1|}}{|\mathbf{r}-\mathbf{R}_1|} dR_1 + \tilde{U}(\mathbf{r}, \mathbf{r}_0, k). \tag{4}$$

Here $\tilde{U}$ is the total noise-field due to both the secondary interactions in D_1 and the scattering (reflection) of waves in the other part of D. This implies that we include in the noise-field anything which cannot be described to a required accuracy (according to the Rayleigh criterion) by the integral term in Eq. (4).

Solution of the integral equation, Eq. (4), is developed in the range of spatial frequencies. Having performed a Fourier transform of the field $U(\mathbf{r}, \mathbf{r}_0, k)$ over $\bar{\mathbf{r}}, \bar{\mathbf{r}}_0$ of the shots and receivers we arrive at

$$\begin{aligned} &U(\boldsymbol{\kappa}, \boldsymbol{\kappa}_0, k) \\ &= F_{\mathbf{r},\mathbf{r}_0}[U(\mathbf{r}, \mathbf{r}_0, k)] = \frac{\Phi(k v_0)}{4} \int c(\boldsymbol{\kappa} + \boldsymbol{\kappa}_0, z_1) \\ &\quad \cdot \frac{e^{-ik[(z_1 - z_0)\sqrt{1-\kappa_0^2/k^2} + z_1\sqrt{1-\kappa^2/k^2}]}}{\sqrt{1-\kappa_0^2/k^2}\sqrt{1-\kappa^2/k^2}} dz_1 \\ &\quad + F'_{\mathbf{r},\mathbf{r}_0}[\tilde{U}(\mathbf{r}, \mathbf{r}_0, k)], \end{aligned} \tag{5}$$

where $\boldsymbol{\kappa} = (\kappa_x, \kappa_y)$, $\boldsymbol{\kappa}_0 = (\kappa_{x_0}, \kappa_{y_0})$ are spatial-frequency variables. In deriving Eq. (5) we make use of the Weyl formula and consider the requirement that $c(\mathbf{R}) = 0$ if $z < 0, z_0$; $[c(\mathbf{R}) = 0$, if $\tilde{v}(\mathbf{R}) = 0]$. Introducing the focusing transformation of the observed field for the plane $z > 0, z_0$ gives us

$$W_0(\boldsymbol{\kappa}, \boldsymbol{\kappa}_0, z) = \int U(\boldsymbol{\kappa}, \boldsymbol{\kappa}_0, k)\, \Psi(\boldsymbol{\kappa}, \boldsymbol{\kappa}_0, k, z)\, dk, \tag{6}$$

in which the function Ψ for the total areal observation has the form

$$\Psi = 2\frac{\sqrt{1-\frac{\kappa_0^2}{k^2}} + \sqrt{1-\frac{\kappa^2}{k^2}}}{\pi \Phi(k v_0)} \Pi\left(\frac{\boldsymbol{\kappa}_0}{k}\right) \Pi\left(\frac{\boldsymbol{\kappa}}{k}\right) \cdot e^{+ik\left[(z-z_0)\sqrt{1-\frac{\kappa_0^2}{k^2}} + z\sqrt{1-\frac{\kappa^2}{k^2}}\right]}, \tag{7}$$

$$\Pi(\mathbf{X}) = 1, \quad |\mathbf{X}| \leqq 1; \qquad \Pi(\mathbf{X}) = 0, \quad |\mathbf{X}| > 1.$$

The presence of the factors $\Pi(\boldsymbol{\kappa}_0/k)$ and $\Pi(\boldsymbol{\kappa}/k)$ indicates that solution of Eq. (4) is in the range of steadily recorded homogeneous plane waves of the full spatial field spectrum. It appears that such truncation of spatial frequencies may not lead to "smearing" of the solution $c(\mathbf{R})$. Integration over k in Eq. (6) is performed in the range of steady recording of temporal frequencies.

Let us turn to spatial-frequency variables $\mathbf{k}_r = \boldsymbol{\kappa} + \boldsymbol{\kappa}_0$, $\gamma = \boldsymbol{\kappa} - \boldsymbol{\kappa}_0$ and Fourier-transform the field $W_0(\mathbf{k}_r, \gamma, z)$ over the coordinate z. As a result of this transform in the full spectral domain $(\mathbf{v}, \gamma)$ $[\mathbf{v} = (\mathbf{k}_r, k_z) = (k_x, k_y, k_z)]$ we obtain (Zherniak, 1982):

$$\begin{aligned} W_0(\mathbf{v}, \gamma) &= F_z[W_0(\mathbf{k}_r, \gamma, z)] \\ &= [c(\mathbf{v}) + \xi(\mathbf{v}, \gamma)]\, \Pi\left(\frac{\mathbf{k}_r \cdot \gamma}{k_z^2}\right). \end{aligned} \tag{8}$$

Here $c(\mathbf{v}) = F_{\mathbf{R}}[c(\mathbf{R})]$ is the 3-D spectrum of the functional-image to be determined, $\xi(\mathbf{v}, \gamma)$ is the spectrum of the transformed noise-field in Eq. (4). Equation (8) shows that the useful part $c(\mathbf{v})$ of the field transformed in this manner does not depend on the variable γ, while the noise-field $\xi(\mathbf{v}, \gamma)$ from $W_0(\mathbf{v}, \gamma)$ remains dependent on it. In the absence of the noise background, $c(\mathbf{v})$ can be defined by $W_0(\mathbf{v}, \gamma)$ at any fixed value of $|\gamma|$, i.e. γ is a free parameter in Eq. (8). This appears to be the consequence of redundancy of the observation system of multifold coverage considered.

One can take advantage of the fact that the spectrum of the solution, $c(\mathbf{v})$, we are seeking is independent of γ to filter out the regular noise-field $\xi(\mathbf{v}, \gamma)$ entering into $W_0(\mathbf{v}, \gamma)$. Suppression of the noise-field can be performed, for instance, by integrating (stacking) $W_0(\mathbf{v}, \gamma)$ over γ with the weighting function $\varphi(\mathbf{v}, \gamma)$ satisfying the requirement

$$\int \Pi\left(\frac{\mathbf{k}_r \cdot \gamma}{k_z^2}\right) \varphi(\mathbf{v}, \gamma)\, d\gamma = 1, \tag{9}$$

which implies that suppression of the noise from $W_0(\mathbf{v}, \gamma)$ must not lead to a distortion of the reconstructed image $c(\mathbf{R})$. Equation (9) appears to have a variety of solutions $\varphi(\mathbf{v}, \gamma)$. For instance, it can be satisfied by functions

$$\varphi = (\mathbf{k}_r)/2k_z^2, \ (\mathbf{k}_r)/[\pi\sqrt{k_z^2 + (\mathbf{k}_r \cdot \gamma)^2}],$$

etc. It allows one to select, among stacking functions φ, those which maximize the signal/noise ratio on reconstructed images. Thus, when selecting φ one can establish the condition of minimization of relative energy of the remaining regular noise, i.e.

$$E_{\text{rel}}\left[\int \Pi\left(\frac{\mathbf{k}_r \cdot \gamma}{k_z^2}\right) \xi(\mathbf{v}, \gamma)\, \varphi(\mathbf{v}, \gamma)\, d\gamma\right] = \min. \tag{10}$$

In real data processing there is, as a rule, a priori information about the composition of regular and irregular noise. Using this information and special techniques for the analysis of the wavefield and reconstructed images, one can select the most effective stacking function φ.

The general form of transformation of the space-time spectrum $U(\mathbf{k}_r, \gamma, k)$ of the observed field to the image $W(\mathbf{R})$ of the inhomogeneous medium in variables $\mathbf{k}_r, \gamma, k, z$ is as follows:

$$\begin{aligned} W(\mathbf{R}) = F_{\mathbf{k}_r}^{-1}[&\iint U(\mathbf{k}_r, \gamma, k)\, \Psi(\mathbf{k}_r, \gamma, k, z) \\ &\cdot \varphi(\mathbf{k}_r, \gamma, k)\, dk\, d\gamma]. \end{aligned} \tag{11}$$

The value of the function φ in variables $\mathbf{k}_r, \gamma, k$ can be derived from its values in the frequency range $\mathbf{v}, \gamma$, if we substitute k_z by

$$k[\sqrt{1-(\mathbf{k}_r+\gamma)^2/4k^2} + \sqrt{1-(\mathbf{k}_r-\gamma)^2/4k^2}].$$

Having interpreted the results of the imaging transformations, it is possible to properly define the structure of the domain D_1 of the inhomogeneous medium where expansion (3) is appropriate. Usually this is the upper part of the medium, lying below the observation system, where the downward source field is described fairly accurately (in phase) by the function $\mathscr{E}_0(\mathbf{R}, \mathbf{R}_0, k)$. The image of the other parts of the medium is, in this case, suppressed and geometrically distorted, as the field re-radiated outside D_1 at this stage of solution refers to the noise background. By means of the visual analysis of the image obtained it is possible, to establish approximately where the lower boundary of the area D_1 is located. This continuation corresponds to a transfer of the entire system of observations below D_1, which makes it possible to correct parameters of the field equation for useful waves, reflected and scattered by deeper inhomogeneities of the medium, to solve a similar imaging problem for the next subregion D_2, and so on.

If the velocity structure of the medium varies strongly in a horizontal direction, images of its domains, which then form the total picture, are reconstructed in the same order.

A limited range of temporal frequencies

We have obtained the solution of the above integral equation, Eq. (4), assuming that $U(\mathbf{r}, \mathbf{r}_0, k)$ is steadily recorded for all $\omega = k v_0$. This assumption disagrees with real conditions in, practically, a finite range of temporal frequencies. The higher the maximum frequency, the smaller the size of any defined region D_n of the inhomogeneous medium must be, according to the Rayleigh criterion used by us. On the other hand, with the decrease of the frequencies in the field being processed, the resolving capability and the details of the images become worse. Varying the range of frequencies, involved in the processing, will make it possible to firstly reconstruct crude, smoothed images of large domains of a medium and then, adding high frequencies, to define more exactly and in more detail the structure of its separate parts.

We take the limitation on the range of frequencies $|\omega| < \Omega$ into account by introducing the characteristic function $\Pi(\omega/\Omega)$ into the imaging transformation. The field in this case is written as follows:

$$U(\mathbf{r}, \mathbf{r}_0, k, K) = U(\mathbf{r}, \mathbf{r}_0, k)\,\Pi(k/K), \qquad K = \Omega/v_0. \tag{12}$$

Applying transformations (5)–(8) to the field $U(\mathbf{r}, \mathbf{r}_0, k, K)$ gives us

$$W_0(\mathbf{v}, \gamma, K) = [c(\mathbf{v}) + \xi(\mathbf{v}, \gamma)]\,\Pi\left(\frac{\mathbf{k}_r \cdot \gamma}{k_z^2}\right)\Pi\left(\frac{k_{1,2}}{2K}\right), \tag{13}$$

where

$$k_{1,2} = \pm\sqrt{k_z^2 + k_r^2 + \gamma^2 + (\mathbf{k}_r \cdot \gamma)^2/k_z^2}.$$

This relation differs from Eq. (8) obtained earlier only by the additional term $\Pi(k_{1,2}/2K)$. Just as in the case of an infinite temporal frequency band, we are free here only to choose stacking functions $\varphi(\mathbf{v}, \gamma, K)$ whose form depends now on the limiting frequency $\Omega = K v_0$. The solution is constructed in a similar way when the spectrum of the field being processed is defined in the limited frequency band $\Omega_{\min} < |\omega| < \Omega_{\max}$. The field W_0 obtained has a form of the difference

$$W_0(\mathbf{v}, \gamma, K_{\max}) - W_0(\mathbf{v}, \gamma, K_{\min}).$$

Analysis of Eq. (13) shows that, in the finite range of temporal frequencies $|\omega| < \Omega$, non-zero values of the spatial spectrum $c(\mathbf{v})$ of the image are determined only inside the sphere $|\mathbf{v}| \leqq 2K$. This allows us to find limiting estimates of the resolving capability of the image $W(\mathbf{R}, K)$. According to the Rayleight criterion, the limiting resolving capability is approximately half the width of the central maximum of the function of the system's response to the field of an elementary radiator of waves. In this case the limiting resolution, uniform over x, y, z, is equal to $0.35\lambda_{\min}$, where $\lambda_{\min} = 2\pi v_0/\Omega$ is the minimum wavelength in the spectrum of the processed field. It is possible to improve the resolution over one of the coordinates x, y, z a little by reducing it over other coordinates in truncation of the respective spatial frequencies.

Other systems of observation

When we solve 3-D seismic problems particular difficulties of technological character arise in the use of the total areal system of observations of the 5-D field $U(\mathbf{r}, \mathbf{r}_0, t)$. The algorithm described for the 3-D imaging inversion can be modified for use in less redundant areal systems of observations of the field over four space-time coordinates. An example of such a system is a set of parallel profiles of multifold coverage. Under the same assumptions, the temporal spectrum of the field, recorded only on straight lines along which its sources shift, will be described by Eq. (4), if $y_0 = y$, $z_0 = 0$. The Fourier transform of the field over the coordinates x_0, x, y of sources, receivers and profiles has the form

$$\begin{aligned} U(\kappa_x, \kappa_{x0}, k_y, k) &= F_{x, x_0, y}[U(x, x_0, y, k)] \\ &= \frac{k^2 \Phi(k v_0)}{4\sqrt{2\pi}} \int \frac{c(\kappa_x + \kappa_{x0}, k_y, z_1)}{\sqrt{z_1}} \\ &\quad \cdot \frac{e^{-i\,\mathrm{sig}(k)[z_1\sqrt{(\alpha+\beta)^2 - k_y^2} - \pi/4]}}{[\alpha\beta\sqrt{(\alpha+\beta)^2 - k_y^2}]^{1/2}}\, dz_1 \\ &\quad + F_{x, x_0, y}[\tilde{U}(x, x_0, y, k)], \end{aligned} \tag{14}$$

where $\alpha = \sqrt{k^2 - \kappa_x^2}$, $\beta = \sqrt{k^2 - \kappa_{x0}^2}$. This relation was obtained using the stationary phase method, the errors of this approach being negligibly small if $k z_1 \gg 1$.

Applying Eqs. (6) and (8) to $U(\kappa_x, \kappa_{x0}, k_y, k)$ and using the function

$$\begin{aligned} \Psi &= \frac{4\,\Pi(\kappa_x/k)\,\Pi(\kappa_{x0}/k)\,\Pi[k_y/(\alpha+\beta)]}{\sqrt{2\pi}\,|k|\,\Phi(k v_0)} \\ &\quad \cdot \frac{(\alpha+\beta)^2\, e^{+i\,\mathrm{sig}(k)[z\sqrt{(\alpha+\beta)^2 - k_y^2} - \pi/4]}}{[\alpha\beta\sqrt{(\alpha+\beta)^2 - k_y^2}]^{1/2}}, \end{aligned} \tag{15}$$

we obtain the full spatial spectrum of the field $W_0(k_x, k_y, \gamma, z)$:

$$W_0(\mathbf{v}, \gamma) = [c_1(\mathbf{v}) + \xi(\mathbf{v}, \gamma)]\, \Pi\left(\frac{\gamma k_x}{k_y^2 + k_z^2}\right). \tag{16}$$

Here $c_1(\mathbf{v}) = F_{\mathbf{R}}[c(\mathbf{R})/\sqrt{z}]$, $k_x = \kappa_x + \kappa_{x_0}$, $\gamma = \kappa_x - \kappa_{x_0}$, $\mathbf{v} = (k_x, k_y, k_z)$. The effect of band-limiting the temporal frequencies here is the same as before.

Thus, the solution of this areal system turns out to be similar to that considered above. This example shows that it is possible to modify the transformations described for the case of various redundant systems. In particular, if the medium's structure does not change in the y-direction (perpendicular to the profile), it is sufficient to have data, recorded only along one profile $y=0$, in order to determine the 2-D imaging inversion. One must only set $k_y = 0$ in Eqs. (14) and (15) and select respective stacking functions.

Continuation of wavefields

Within the framework of a dynamic imaging problem the inverse continuation of the field is a means to continue the entire observation system outside the reconstructed domain of an inhomogeneous medium. This implies that one must compute the values of such a field whose "sources" and "receivers" would be at some surface below the reconstructed part D_1 of the medium and the homogeneous domain would be above the surface. In this case it is sufficient for the field to consist only of homogeneous plane waves, i.e. to have no peculiarities in its secondary "sources". Thus, in continuation of the observed field it is necessary that the influence on it of the known inhomogeneities of the upper part of the cross-section be compensated for as accurately as possible.

We can use the procedures of continuation of the field in a hypothetical inhomogeneous medium in the finite-difference migration algorithms based on various approximate solutions of the wave equation (Claerbout, 1976; Kosloff and Baysal, 1983). However, in the case of such field continuation, the attenuation of wave amplitudes due to their travel through inhomogeneities is not compensated for. The amplitude distortions increase due to the inverse travel of waves through the inhomogeneities.

Zherniak (1983) proposed an algorithm for a recurrent local application of Kirchhoff's integral formula to continue the field in a spatially inhomogeneous medium. The structure of a medium is approximated by a set of thin phase variable-velocity plates. In the case of consistent continuation of the field through the plates we consider only phase variations of homogeneous plane waves from its spatial spectrum. The approximations and the form of the algorithm are in agreement with the approximations and the form of the imaging inversion. The local phase continuation of the field allows us to take into consideration wave refractions in an inhomogeneous medium and partially compensate for an undesirable attenuation of their amplitudes. This, in fact, appears to be a specific method of regularization of the inverse problem in the field reconstruction.

Let us represent the formula of phase continuation of the field $U(\mathbf{r}, z, \omega)$ from the upper to the lower boundary of an inhomogeneous thin layer $(z, z+\Delta z)$ in the space-frequency domain as follows:

$$U(\boldsymbol{\kappa}, z+\Delta z, \omega) = \sum_m \left\{[U(\boldsymbol{\kappa}, z, \omega) \underset{\kappa}{\circledast} S_m(\boldsymbol{\kappa}, z)]\, e^{i\Delta z \omega \sqrt{\frac{1}{v_m^2} - \frac{\kappa^2}{\omega^2}}}\right\}, \tag{17}$$

where $U(\boldsymbol{\kappa}, z, \omega) = F_{\mathbf{r}}[U(\mathbf{r}, z, \omega)]$, S_m is the spatial spectrum of the characteristic function of the m-th plate of the layer components $(z, z+\Delta z)$, v_m is the velocity in it. The distortions resulting from the approximate formula, Eq. (17), are insignificant if $\Delta z \left|\frac{\omega}{v_m} - \frac{\omega}{v_n}\right| \ll 1$ for any m and n. Under continuous vertical and horizontal variations of the velocity in a medium, Eq. (17) becomes:

$$U(\boldsymbol{\kappa}, z+\Delta z, \omega) = \int_{z=\text{const}} U(\mathbf{r}, z, \omega)\, e^{i\left[\Delta z \omega \sqrt{\frac{1}{v^2(\mathbf{r}, z)} - \frac{\kappa^2}{\omega^2}} - \boldsymbol{\kappa}\cdot\mathbf{r}\right]}\, dx\, dy. \tag{18}$$

Such downward continuation of the field, accomplished simultaneously over the coordinates of both the receivers $(\mathbf{r} \to \boldsymbol{\kappa})$ and the sources $(\mathbf{r}_0 \to \boldsymbol{\kappa}_0)$, corresponds approximately to transferring the entire system of observation below the inhomogeneous domain of the medium with partial removal of the effect of upper inhomogeneities. Combined with the imaging transformations, this allow us to organize an interaction process, a means for a continuous and more precise definition of the structure of a highly inhomogeneous medium.

To continue the field in the inhomogeneous medium, we must know at least the mean (interval) values of the velocity in it. These values might be obtained from a well log or from velocity analysis methods, analogous to the "velocity spectrum" algorithm in the CDP method. It appears that maximum "brightness" of the image elements reconstructed by means of imaging transformations can be obtained if the velocity at which they are reconstructed coincides with the actual interval velocity of the domains of the medium. So we can firmly obtain the mean velocity values in any domain by some selection method.

Results of numerical experiments

Possibilities of the algorithms of the inverse dynamic imaging were tested by means of numerical modelling (Alekseev and Zherniak, 1983). A model of a 2-D medium and observation system is shown in Fig. 1. Arrangement of the 64 receivers here was the same for any of the 64 positions of the source of cylindrical waves. Forward dynamic problems were solved by the accurate method of boundary integral equations (Voronin, 1978). Two of the 64 common receiver trace gathers, where direct events were excluded, are shown in Fig. 2. The computed wavefield $U(x, x_0, t)$ was decomposed into Fourier harmonics with respect to the discrete variables $t, x_0, x \to \omega, \kappa_0, \kappa$.

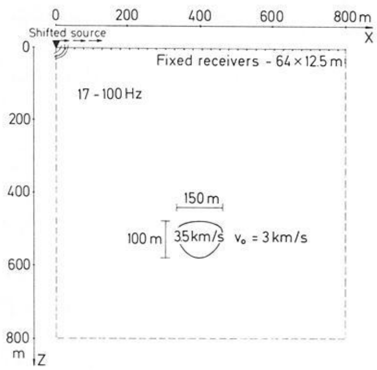

Fig. 1. Model of the medium and the observation system

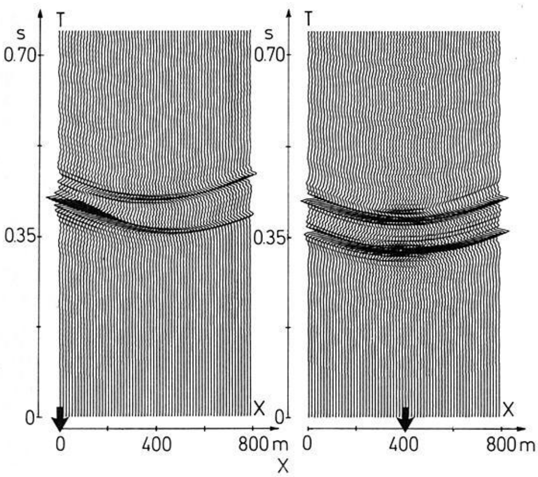

Fig. 2. Two of the 64 common receiver trace gathers for the model in Fig. 1

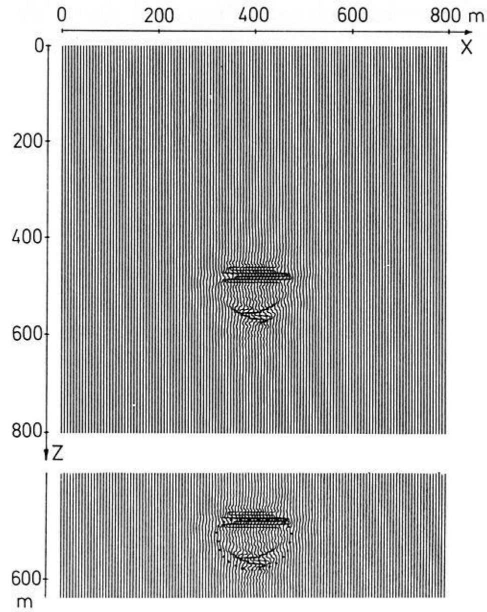

Fig. 3. Image of the object in the first approximation

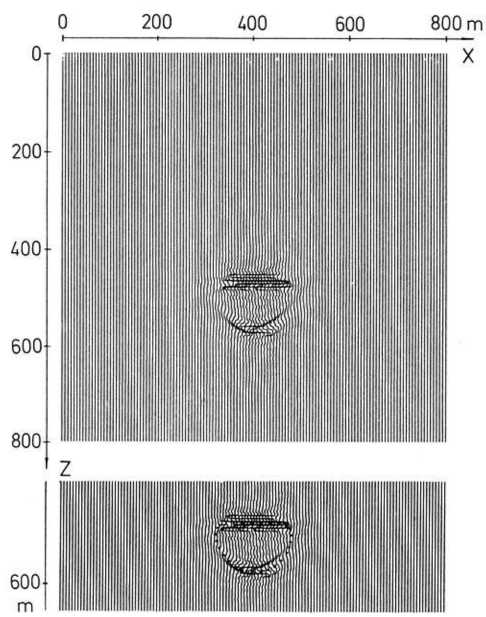

Fig. 4. The result of imaging the structure of the medium in the second approximation

The inverse imaging problem was solved in an interactive regime. In the first approximation, the medium was assumed to be homogeneous. The velocity in it was taken equal to that near the sources. The reconstruction of the image $W(x, z)$ was carried out by means of the 2-D variant (for cylindrical waves) of Eq. (11). The focusing function, Eq. (7), with $\kappa=\kappa_x=(k_x+\gamma)/2$, $\kappa_0=\kappa_{x_0}=(k_x-\gamma)/2$, $z_0=0$ was used. $\varphi \sim 1+5\sin^2[\pi(k_x/2k)^2]$ was chosen to be a rough approximation of the weighting function of γ-stacking. Stacking over 64 values of $k=\omega/v_0$ was performed in the frequency range $17<|\omega/2\pi|<100$ Hz. Only the upper part of the reconstructed image corresponds to the real position of the object (indicated by dots in the lower part of Fig. 3).

After that, the information obtained about the object configuration and the velocity in it was used. The medium was imaged together with double field continuation under the chosen upper part of the curvilinear boundary of the object. For the downward continuation of the field we used Eq. (17) with $m=0, 1$; $v_0=3$, $v_1=3.5$ km/s. At every step Δz through the depth of the object the computation of $U(\kappa, \kappa_0, z, \omega)$ was carried out by means of inverse and direct Fourier transforms over $\kappa \to x$, $\kappa_0 \to x_0$. We substituted the continued field $U(\kappa, \kappa_0, z, \omega)$ in Eq. (11) in place of the field observed on the line $z=0$, and took $z=z_0=0$ in the Ψ-function of Eq. (7). The resulting depth cross-section is shown in Fig. 4. The image of the steep parts of the closed boundary was cancelled out because the rays reflected from them do not strike the fixed arrangement of the receivers.

This undesirable effect can be removed if we use conventional offset-spread systems shifting over long distances. An example of the image reconstruction in this case is shown in Fig. 5. An object here is a sloping boundary with two small scattering inclusions above it. The structure of the medium here does not vary across the profile, $v_0=3$ km/s, $v_1 \ll v_0$, the size of the scattering elements is $\sim\frac{1}{4}\lambda_{\min}$. The source radiates spherical waves in the frequency range 8–64 Hz. With the step

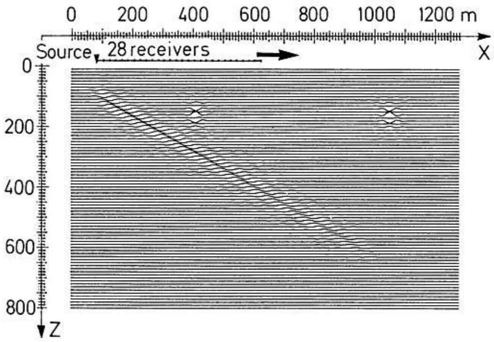

Fig. 5. Image of the 2-D structure of the medium reconstructed on the data of a shifting offset spread

$\Delta x_0 = \Delta x = 20$ m, the source together with 28 receivers shifts along the profile over a distance of 1,200 m. The reconstruction of the image was carried out directly by *Eq.* (11). *The focusing function, Eq.* (15), *with* $k_y = 0$ was used with the same approximation for φ. Similar experiments were also performed for simple 3-D models. The wavefield in this case was observed on a multifold system of independent profiles.

Conclusion

The imaging transforms, resembling to some extent the spectral form of the CDP-migration transformations (Zherniak, 1984) can be looked upon as their dynamic generalization. In comparison to the CDP-migration, with their help one can solve a wider range of seismic problems used when both steeply dipping reflecting boundaries and scattering elements are present in a medium. The choice of the optimum stacking functions guarantees a high signal/noise ratio in connection with the reconstruction of two- and three-dimensional images in seismic sections. Taking the dynamics of reflected and diffracted waves in the observed field more accurately into consideration allows us to obtain a higher resolution in images and to improve the mapping of the physical properties of an inhomogeneous medium. The efficiency of the imaging transformations was tested in a series of numerical experiments.

The solutions of the problem of reconstruction of functional-images obtained can be looked upon as a particular case of quite a general approach to the solution of dynamic inversion problems, using data of overdetermined observation systems. One may choose other equations of the useful part of the field to formulate problems of reconstruction of other stable functionals of the structure of the medium. Similar problems may be posed, for instance, for the equations of elasticity theory. In this case, in order to develop the solution one has to find such transformations of the redundant field as a result of which the useful part of the transformed field (the part corresponding to a formula of the signal component of the field) would be independent of a spatial or a spatial-frequency coordinate. All this makes it possible to find procedures of specific filtration of irregular noise, which do not distort the dynamics of useful signals, from which the structure of the inhomogeneous medium will be determined.

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