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Extremal inversion of vertical displacements, Long Valley Caldera, California 1982/1983

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Abstract. Vertical displacement data from August 1982 and August 1983 leveling surveys in Long Valley Caldera are examined relative to a 1975 datum. These uplifts are hypothesized to be due to the inflation of a magma chamber of arbitrary shape at depth. Using extremal inversion techniques, which allow for uncertainties associated with random survey error, bounds on the depth to the top of the body and on the location of the edges of the body are produced. These bounds are unique horizontal and vertical limits on any possible volume source satisfying the data. The bounds indicate that any volume source satisfying the 1975–1982 leveling data must lie, in part, at or above 12 km. For the 1975–1983 displacement field, some volume change must have occurred at or above 11 km. The east-west bounds on the source have not changed from 1982 to 1983, requiring volume change east of 118.93° W and west of 118.90° W. However, the north-south bounds, which require a portion of the model to lie north of 37.65° N and south of 37.67° N, have widened one grid element to the north during this interval. These estimates are independent of both Poisson's ratio and the exact boundaries of the region modeled.

Key words: Positivity constraints – Inverse problem – Uplift

Introduction

Long Valley Caldera has been the site of recent permanent and seismic displacements. Repeated leveling surveys between 1975 and August 1983 within the caldera have measured up to 0.4 m of vertical displacement. These displacements have contributed to the hypothesis that a magma chamber still exists beneath the caldera and that this chamber has reinflated to some extent. This notion is compatible with recent moment tensor inversions of seismic data (Julian, 1983) and *P*-wave delay-time inversions (Steeples and Iyer, 1976). Similarly, a study of seismic attenuation within the caldera (Sanders and Ryall, 1983) suggests the presence of a "region of molten or partially molten magma".

If one accepts the possibility of a magma body at depth, it is possible to invert the uplift data for parameters of the causative body such as the depth and the volume change. For example, Savage and Clark (1982) inverted the 1982 displacement data of the survey line along Highway 395. Assuming a point source, these authors produced an estimate of the source depth as well as the volume change. Similarly, Castle et al. (1984) inverted the 1983 vertical displacement data along this line for estimates of the same parameters. Recently other models have been proposed. Savage and Cockerham (1984) were able to reproduce observed horizontal and vertical surface deformation reasonably well using two separate dike injection models. The first model consists of a single dike that dips 30° northward beginning at a depth of 8 km and extending to about 12 km in depth. The second model is similar to the first with the addition of a dike extending vertically from the top of the dipping intrusion to within 3 km of the surface. Right lateral slip was needed in both models in order to satisfy the horizontal displacement data. Recently, Rundle and Whitcomb (1984) proposed an additional model. In their model, deformation is attributed to the inflation of two spherical magma chambers; one at a depth of 5 km located 1.5 km west of station Casa (Fig. 1), the other 9 km deep about 5.5 km north-northwest of Casa. All of the above models fit the data reasonably well.

Given the deformation data alone, there is no reason to prefer one model over another. Even when including other information such as gravity or magnetotelluric data in the inversion, some ambiguity will remain in the description of the source. Therefore any proposed model must be viewed critically. Answers to the question "What magma body has produced these displacements?" are seldom unequivocally found. Definite answers are more forthcoming if one asks "How do the data constrain the range of possible models?" One way to answer this is to examine all the models which fit the data and determine properties common to all these models. However, this is a laborious task. There is a method available which allows one to find bounds or limits on certain properties of the models. Limits are placed on model properties such that all models satisfying the data must have properties within these bounds. Such limits are important in allowing one to assess the ambiguity present in the data set. It is for this reason that I have chosen to examine the bounds which the 1975–1982 and 1975–1983 leveling data place on the vertical and lateral extent of a proposed magma body under Long Valley Caldera. Using the method of extremal inversion (Parker, 1975; Sabatier, 1977a, b, c). I derive unique bounds on certain properties of the assumed source. Specifically, the bounds constrain the depth and horizontal extent of the perturbing body. Full nonsymmetric, three-dimensional bodies are allowed and random leveling errors are incorporated into the inversion procedure.

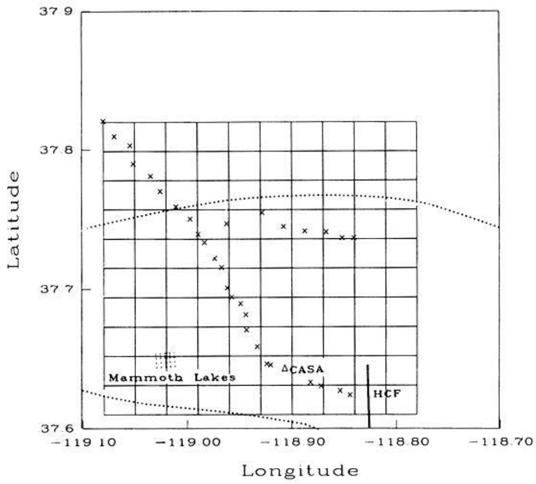

Fig. 1. Map of the Long Valley Caldera region. The caldera is denoted by the *dotted line*. The town of Mammoth Lakes is shown. The Hilton Creek fault is denoted by the *heavy black line* labeled HCF. ×'s represent the leveling stations used in the study. Station Casa is the *labeled triangle*. Lee Vining is off the *upper left hand corner* of the map. The discretization of the region used in the inversion is shown

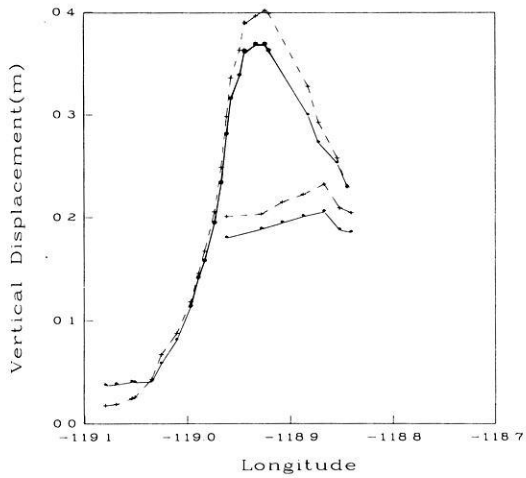

Fig. 2. The vertical displacement data from the August 1982 survey (*solid line*) and the 1983 survey (*dashed line*) projected onto the east-west axis. The data are shown relative to a 1975 baseline

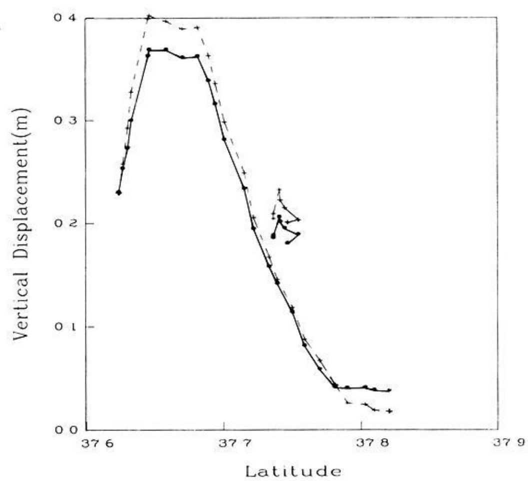

Fig. 3. The vertical displacement data from the August 1982 survey (*solid line*) and the 1983 survey (*dashed line*) projected onto the north-south axis. Again, these data are relative to the 1975 data

The 1982 and 1983 leveling surveys

Leveling surveys within Long Valley Caldera were run along Highway 395 in 1932, 1954, 1975, 1980, 1982 and 1983 (Castle et al., 1984). In addition, surveys were run along various access roads in the area. Figure 1 shows the stations used in the inversion. These are not all of the available data; some data near to and east of the Hilton Creek fault were not used in the inversion. Movement along this fault occurred in May 1980 and displacements, not associated with volume change, would adversely affect the inversion. Furthermore, trilateration data measuring horizontal length changes of survey lines within the caldera were not examined. These data would provide additional constraints on the model parameters.

The early surveys prior to and including the 1975 leveling line detected little or no uplift. However, between 1975 and 1982, up to 0.25 m of uplift occurred. Subsequent surveys in August 1982 and August 1983 detected 0.35 m and 0.40 m of maximum uplift, respectively. This suggests that one may take the 1975 elevation as a baseline with which to measure the changes occurring in the 1975/1982 and 1975/1983 intervals. These elevation changes are shown in Fig. 2 projected onto an east-west trending plane. In Fig. 3 the data are projected onto a north-south trending plane. The data shown are a portion of the leveling lines which extend along Highway 395 from the northwest to the Hilton Creek fault in the southeast. In addition, a second line of data extending approximately east-west was included in the inversion. One assumption made in the production of this uplift profile is that the southern end of the leveling line has remained stable with respect to Lee Vining in the north (Castle et al., 1984), which permitted one to treat the Long Valley system as if it were isolated from the surrounding region. Furthermore, Castle et al. (1984) also argue that only random errors are significant in the data, i.e. systematic deviations were shown to be negligible.

The method of extremal inversion

As mentioned previously, a variety of models have been proposed for the magma body giving rise to the observed displacements. Unfortunately, the data do not allow for discrimination among the various models. For this reason an alternative approach was taken. Properties common to all models fitting the data were searched for. Specifically, bounds on the depth and the horizontal location of the source were found. The method of extremal inversion was used to derive these bounds. This is a versatile technique and one that enables the inversion of all static displacement data, horizontal as well as vertical, to give unique bounds on properties of the source model. The only assumptions made are that the fractional volume change is of one sign and that the region is homogeneous and may be characterized by a single Poisson's ratio. The technique is discussed by Parker (1975), Sabatier (1977a, b), Safon et al. (1977) and Rietsch (1978). The adaptation of this method to the inversion of static earth displacements is given in Vasco and Johnson (1985).

As an introduction to the method consider the discrete case. The region of interest is divided into N blocks. Each block is capable of undergoing some fractional volume change $\Delta\vartheta$. A linear system of equations relates the fractional volume changes undergone by the blocks to the M displacements m_i measured at the surface,

$$\sum_{n=1}^{N} k_{i,n}\Delta\vartheta_n = m_i \qquad i=1,2,3,\ldots,M. \tag{1}$$

There is also the requirement that all fractional volume changes are of one sign, in this case they are non-negative,

$$\Delta\vartheta_n \geq 0 \qquad n=1,2,3\ldots,N.$$

Here $\Delta\vartheta_n$ denotes the fractional volume change within the subregion ω_n, m_i denotes the measured displacement and $k_{i,n}$ gives the deformation at the ith station due to a unit fractional volume change within the nth block. $k_{i,n}$ is given by the point source response at the ith station integrated over the nth block,

$$k_{i,n} = \int_{\omega_n} K_i(\xi)\, dV(\xi).$$

Here $dV(\xi)$ denotes a volume element of the nth block and $K_i(\xi)$ denotes the integrand relating the displacement at station x^i to the fractional volume change at a point ξ. For vertical displacements,

$$K_i(\xi) = \frac{1}{3\pi}(\nu+1)\frac{\xi_3}{S^3}$$

where

$$S = \sqrt{(x_1^i-\xi_1)^2+(x_2^i-\xi_2)^2+\xi_3^2}.$$

ν is Poisson's ratio for the half space.

The general discrete extremal linear inversion problem is to find the extremum, minimum or maximum, of a generalized moment

$$A = \sum_{n=1}^{N} \alpha_n \Delta\vartheta_n \tag{2}$$

subject to the constraint that Eq. (1) is satisfied, i.e. the data are satisfied. The α_n in the above equation are constants. The generalized moment A may represent some physically significant property depending on the possible choices of the constants α_n. For example, choosing

$$\alpha_n = |\xi-\xi_0|^k \Delta V_n,$$

where ΔV_n is the volume of the ω_n subregion, will produce a bound on the kth-order moment about the point ξ_0. For $k=0$ a bound on the total volume change in the region is produced,

$$A = \sum_{n=1}^{N} \Delta V_n \Delta\vartheta_n.$$

For $k=1$,

$$A = \sum_{n=1}^{N} |\xi-\xi_0|\, \Delta V_n \Delta\vartheta_n$$

the moment of the body about the point ξ_0 is given. Volume changes further from ξ_0 are given greater importance than changes closer to this point. Hence, A in this case can be considered as a measure of the compactness of the body much like the moment of inertia.

Another possible choice of α_n and the one used in the following application to the Long Valley uplift data is

$$\alpha_n(\xi) = \begin{cases} 1 & \omega_n \text{ in } U \\ 0 & \omega_n \text{ not in } U \end{cases} \tag{3}$$

where U is a region of interest, that is some subset of the N blocks. Using such α_n, A represents the total fractional volume change in the region U. Such a choice can be used to bound the extent of the causative body in the following manner. If the lower bound of the linear functional A is zero then there exists at least one possible body which satisfies the data but has no volume change in the specified region U. On the other hand, if the smallest value of A is not zero then some volume change must have occurred in U.

The method of extremal inversion may be modified to treat data containing random errors ε_i. In this case one is interested in minimizing the moment (2) subject to the inequality constraints

$$m_i - \varepsilon_i \leqq \sum_{n=1}^{N} k_{i,n}\Delta\vartheta_n \leqq m_i + \varepsilon_i \qquad i=1,2,3,\ldots,M \tag{4}$$

$$\Delta\vartheta_n \geqq 0 \qquad n=1,2,3,\ldots,N.$$

This problem may be transformed into one of the form (1) (Hadley, 1962). The values of ε_i can be estimated for the above surveys. It has been argued (Castle et al., 1984) that the errors in the leveling survey are principally random errors described by the standard deviation

$$\sigma_i = \gamma L_i^{\frac{1}{2}}$$

where γ is a constant and L_i is the distance between the ith station and the base bench mark. For the single-run first-order leveling surveys of 1982 and 1983, $\gamma = 2.0$ mm/km$^{\frac{1}{2}}$. For the double-run first-order leveling survey of 1975, $\gamma = 1.5$ mm/km$^{\frac{1}{2}}$. The lower precision value $\gamma = 2.0$ mm/km$^{\frac{1}{2}}$ was considered as a measure of the error in the 1975–1982 and 1975–1983 data. Assuming a Gaussian distribution of errors, the 95% confidence intervals for $m_i(\pm 2\sigma_i)$ were incorporated into the inversion.

Application of extremal inversion to the leveling data of Long Valley Caldera

The method of extremal inversion was applied to the data set discussed above. The aim was to determine bounds on the depth to the top of the body and on the location of the east-west and north-south boundaries of all possible magma bodies fitting the data. First the region of interest was divided into 15 layers and each layer was divided into 100 horizontal blocks. This resulted in 1500 blocks, each of 1 km height, 2.65 km east-west length, and 2.39 km north-south length. The initial volume of each block was 6.33 km^3. The range of possible models then is represented by all the possible combinations of fractional volume changes within the blocks. Given this space of possible models and a desire to produce bounds on the depth and horizontal boundaries of acceptable models, it is necessary to define an appropriate generalized moment. I have chosen to use α_n as defined in Eq. (3).

First examine the bound on the depth to the top of

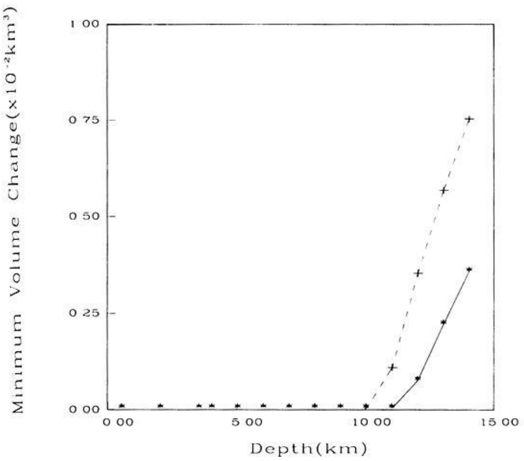

Fig. 4. Vertical depth bounds derived from the leveling data in Fig. 2. Shown here is the minimum volume change which must occur above the given depth. The bounds derived from the 1982 data are denoted by a *solid line*, while the 1983 bounds are denoted by a *dashed line*

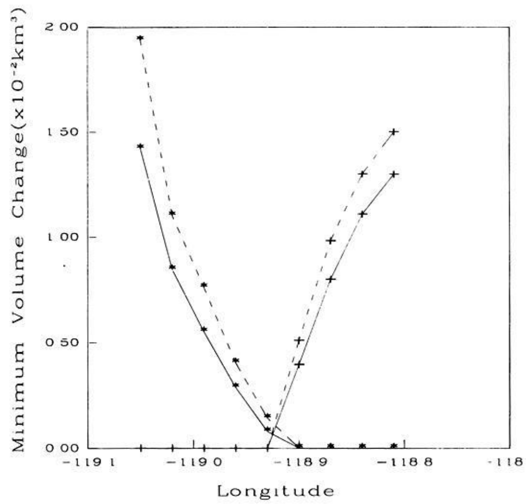

Fig. 5. East-west (longitudinal) bounds derived from the leveling data. This displays the minimum volume change which must occur to the east of the points* and to the west of the points +. The bounds on the 1982 data are indicated by *solid lines*, while the 1983 bounds are given by *dashed lines*

the magma body. Consider a horizontal plane which lies at a depth h_1. Define U to be the region between h_1 and the free surface and find the solution $\Delta\vartheta_n$, $n=1,2,...,N$ which minimizes the total fractional volume change in the region U given by the functional A while still satisfying the constraints (4). Now move to a greater depth h_2 and repeat the process. For each depth $(h_1, h_2, ...)$ one has a particular minimum value of A. Plotting these particular minimum values of A derived for the various regions with lower boundaries given by h_i against the depth h_i results in the curves in Fig. 4. The least upper bound on the depth of the body is given by the depth of the first point where the volume change is nonzero, for this is the shallowest depth above which some volume change is required in order to satisfy the data. If the lower boundary of the region U extends down to or deeper than this point, then some volume change is required in U. The lower bound on the required volume change is given by the ordinate. As can be seen in Fig. 4, in order to satisfy the 1982 leveling data some volume change must have occurred above 12 km. For the 1983 survey the bound is 11 km.

The method of extremal inversion was also used to produce horizontal bounds on the body. This was done in the same way as for the vertical bounds. A plane perpendicular to a specified direction defines a region U to the right or to the left of the plane. The minimum volume change in region U is sought and the plane is then shifted to a new position. The results are shown in Figs. 5 and 6 for east-west and north-south directions, respectively. Here, both right and left bounds are shown. In Fig. 5, one can see that there has been essentially no change in the east-west bounds between August 1982 and August 1983. However, there has been a change in the north-south limits to the body. The bound for the 1983 data has moved one grid element (2.39 km) to the north.

Discussion and conclusions

Extremal inversion techniques were able to produce depth bounds on a proposed magma body within Long Valley

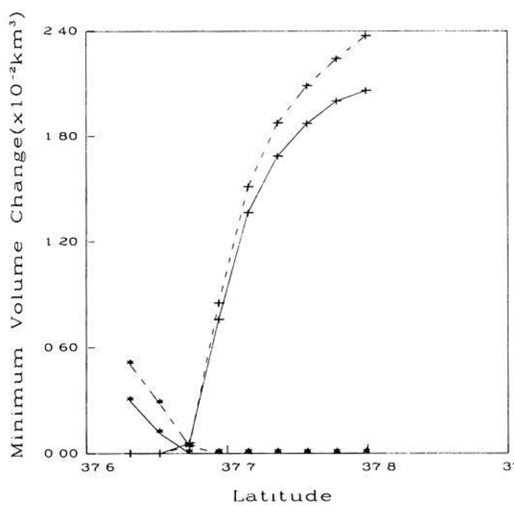

Fig. 6. North-south bounds (latitudinal) derived from the leveling data. This displays the minimum volume change which must occur to the north of the points* and to the south of the points +. The bounds on the 1982 data are indicated by *solid lines*, while the 1983 bounds are given by *dashed lines*

Caldera. The August 1982 leveling data require a volume change above a depth of 12 km while the August 1983 data require some volume change above a depth of 11 km. The horizontal bounds are for the most part unchanged, the only difference is the northward extension of the north-south bounds by one grid element. The significance of these results lies in what they indicate about the range of models that may fit the vertical leveling data. The sole conclusion one may make about the depth to the top of any supposed magma chamber is that it must be less than or equal to 12 km in 1982 and less than or equal to 11 km in 1983. As for the horizontal bounds (Figs. 5 and 6), in the case of the longitudinal or east-west bounds the only requirement is that volume change occur east of 118.93° W and west of 118.90° W. Similarly, any model satisfying the data for both 1982 and 1983 must lie north of 37.65° N and

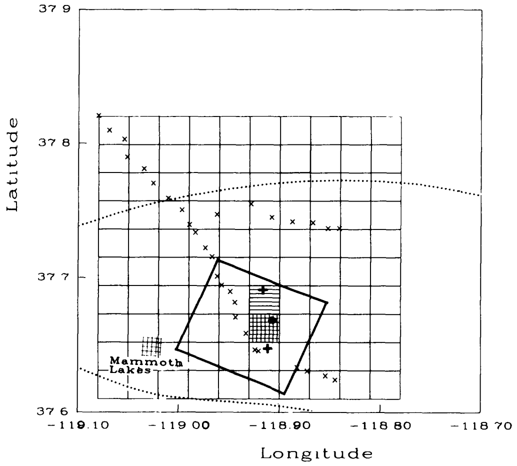

Fig. 7. Map of the Long Valley/Mono Craters region showing the 1982/1983 survey lines used. Stations are denoted by ×'s. The horizontal discretization of the region is indicated. The town of Mammoth Lakes is shown, as is the caldera boundary (*dotted line*). The horizontal bounds for 1975–1982 are shown (*vertical hatches*), as are the bounds for 1975–1983 (*horizontal hatches*). The point source model of Castle et al. (1984) is denoted by *the dot*. The two magma chambers of Rundle and Whitcomb (1984) are denoted by *crosses*. The projection of the dipping dike model of Savage and Cockerham (1984) on to the surface is the *black rectangle*

south of 37.67° N. The 1983 north-south bounds differ in that the northernmost bound has moved outward to 37.69° N. For the discretization given it is not possible to constrain the width of the source body. If one assumes that a homogenous body has given rise to the data then the body must lie, in part, between 118.93° W and 118.90° W and between 37.65° N and 37.67° N. So the geometrical constraints on possible models satisfying the data have been clearly laid out. It should be noted that these are necessary and not sufficient constraints. Any model satisfying the data must include some volume change in the region described above, but a model with volume change in the region does not necessarily satisfy the data. Finally, the bounds derived are not merely the properties of a point source in the given model space (discretization). This is because it is the requirement that the models fit the data within two standard deviations which determines the bounds.

A number of models have been proposed to explain the vertical and horizontal deformation (Rundle and Whitcomb, 1984; Savage and Cockerham, 1984; Castle et al., 1984). Though the models do differ in detail they seem to share some common properties. All models require inflation of a magma reservoir beneath the resurgent dome to fit the vertical displacement data. It is interesting to compare the above models with the bounds placed on the range of possible models by the method of extremal inversion. First consider the point source model of Castle et al. (1984). For the 1975–1983 vertical deformation data their model lies within the specified extremal bounds (Fig. 7). Similarly, the models of dipping dike intrusions of Savage and Cockerham (1984) satisfy the horizontal bounds (Fig. 7). Finally, consider the model of Rundle and Whitcomb (1984) which involves the inflation of two spherical magma chambers, one at a depth of 5 km located 1.5 km west of leveling station Casa and the other 9 km deep about 5.5 km northwest of Casa. Their two magma chambers taken separately do not satisfy the horizontal bounds derived above. However, because the body is not a single body it is not required to lie in the region shown in Fig. 7, the intersection of the horizontal bounds. Non-convex or multiply connected bodies can satisfy the bounds without having volume change occur within the region in Fig. 7. The depths to the model of Rundle and Whitcomb of 5 and 9 km also satisfy the restriction that some or all of the magma intrusion occur at or above 12 km in the 1975–1982 interval and at or above 11 km in the 1975–1983 interval. Furthermore, the models of Castle et al. (1984) and Savage and Cockerham (1984) also satisfy the depth bounds derived. Therefore, the extremal bounds encompass three recently proposed models. However, the importance of the extremal bounds is not in judging proposed models. The importance lies in what the method states about the limits of the vertical displacement data in determining the location and shape of a model. The best one can say with the given data set, for the chosen parameters, is that volume change must have occurred somewhere in the rectangle defined by the latitude, longitude and depth bounds if the body is assumed to be a single convex body. If multiple or non-convex bodies are allowed, then one can merely say that the bodies must be distributed such that all of the individual bounds are satisfied.

In addition to the vertical and horizontal displacement data, the models are constrained by gravitational and magnetic field changes, teleseismic *P*-wave residuals (Steeples and Iyer, 1976) and *S*-wave attenuation data (Sanders, 1984). One might hope that comparisons could be made among the various data sets. Extremal inversion can provide one model-independent way to accomplish this. Extremal inversion techniques have been developed for gravity and magnetic (Safon et al., 1977), temperature (Huestis, 1979) and electro-magnetic induction (Weidelt, 1981) problems as well as for static displacements. The bounds derived from each of the above data sets can be compared. The data set which most tightly constrains some model property such as the depth to the top of the magma body can be determined. So the effectiveness of each data set in constraining the range of possible models becomes clear. This allows a more realistic assessment of the constraints on the body giving rise to the data sets.

It must be pointed out that the analysis was somewhat simplified. It was assumed that all fractional volume changes were positive. This excludes local deflation and assumes that the source of the material causing the expansion was sufficiently removed from the stations. Also, the interpretation of the bounds depends on assumptions of the form of the body, i.e. if it is convex or non-convex. A Poisson's ratio of 0.25 was assumed for the whole caldera. But, as can be shown, Poisson's ratio does not affect the depth estimate itself, though it does affect the minimum volume change estimates. Also, although it passed nearby, the survey did not traverse the region of maximum uplift. Hence, the depth bounds are slightly deeper than necessary but are still valid and unique for the given data set. A homogeneous halfspace was assumed in the calculations. Jovanovich et al. (1974) noted the effects of layering upon displacements. They also presented integral relationships between displacement and volume change which would allow one to invert the leveling data while accounting for overlying structure. Layers of high rigidity, such as thick lava flows, tend to reduce and broaden surface flexure resulting in an overly cautious depth bound and a greater minimum width estimate. Finally, the volume change was assumed to have taken place in a specified region. That region was then discretized. By changing the boundaries of the volume considered, it was found that the extremal bounds are insensitive to the exact extent of the region. For example, vertical bounds were calculated for regions with total depth extents of 15, 20 and 25 km. The minimum volume changes at depth were identical for each of the regions. In order to estimate the depth or width bounds correctly it is only necessary for the region considered to encompass the boundary between the area of zero volume change and the area of nonzero volume change. However, the exact location of the bounds depends on the discretization; finer divisions of the region will give better bounds. One is only limited by computational expense in deciding on a discretization. Given these caveats, I believe that this is a robust technique which has produced meaningful bounds on the source volume in Long Valley Caldera.

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