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The influence of a dilatant region in the Earth's crust on the Earth tide tilt and strain*

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Abstract. A model is presented to account for the influence of a buried dilatant 3-*D* ellipsoidal inclusion on the Earth tide tilt and strain. The additional Earth tide effects due to the inclusion thus obtained are here referred to as dilatant zone tidal effects (DZE). Magnitude and phase shift of these effects are determined by the geometrical properties of the inclusion and the contrast between the elastic parameters in the inclusion and in the surrounding material. As an application, tidal effects of dilatant earthquake preparation zones are calculated using observed travel-time delays of seismic waves. So the dilatant zone of the $M = 2.6$ Blue Mountain Lake earthquake leads to a DZE tidal tilt of more than 300% of the undisturbed tidal tilt. This suggests that tidal observations are a viable tool for earthquake prediction, especially for shallow earthquakes with focal depths of less than 10 km.

Key words: Earth tides – Dilatancy – Earthquake prediction

Introduction

Beaumont and Berger (1974), in their pioneering work using finite element methods, showed that an amplification of the Earth tide tilts and strains of up to 60% could be expected by a buried dilatant inclusion in the Earth's crust (Fig. 1). Similar results were obtained by Molodensky (1983) who found an analytical solution of this problem. This led to the expectation that measurements of Earth tide tilts could be a very useful tool in earthquake prediction (Zschau, 1979). But all these calculations were made by two-dimensional models and, therefore, the coupling effect of the third component of the tidal strain had to be neglected. Hence, no phase shift between the regular Earth tide field and the induced tidal field due to the ellipsoidal inclusion was obtained. In order to avoid this disadvantage of two-dimensional models an attempt is made to construct a three-dimensional model, in order to calculate the tidal field induced by dilatant zones. This model is based on Eshelby's (1957) solution of the problem of calculating the strain field in the vicinity of an ellipsoidal inclusion in a homogeneous body.

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Construction of the model

Consider an ellipsoidal inclusion with semi-axes a, b, c and located at depth z (all dimensions are in km, Fig. 2) of the Earth's crust, having elastic properties different from those of the surrounding material. What will be the influence of this body on Earth tide tilt and strain observed at a point (x, y) on the surface in the vicinity of the inclusion? To answer this question we calculate the amplitude of the additional tilt and strain at the point (x, y) produced by the inclusion under the influence of the Earth tide strain field. The additional tidal effects will be referred to here as the dilatant zone tidal effects (DZE). The actual tidal field measured at (x, y) is obtained by vector addition of the regular and the DEZ tidal field.

The semidiurnal tidal potential as presented by Laplace is given (Melchior, 1978) by

$$W = \frac{3}{4} G \cdot M \cdot \frac{r^2}{d^3} \cos^2\theta \cos 2\vartheta, \tag{1}$$

G = gravity constant,
M = mass of the tide-generating celestial body (in the equatorial plane),
r = radius of the Earth,
d = distance between the centres of mass of the Earth and the celestial body,
θ = latitude of the observation point,
ϑ = hour angle of celestial body with respect to the Greenwich meridian.

Assuming the conditions of a free surface (this assumption is valid to great depths due to the long wavelength of the semidiurnal tides), the tidal strain is given by

$$\begin{aligned}
\varepsilon_{11} &= \varepsilon_{\theta\theta} = \frac{l}{g \cdot r} \cdot \frac{\delta^2 W}{\delta\theta^2} + \frac{h}{g \cdot r} W, \\
\varepsilon_{22} &= \varepsilon_{\vartheta\vartheta} = \frac{l}{g \cdot r \cdot \sin^2\theta} \frac{\delta^2 W}{\delta\vartheta^2} \\
&\quad + \frac{l \cdot \cos\theta}{g \cdot r \cdot \sin\theta} \frac{\delta W}{\delta\theta} + \frac{l}{g \cdot r} W, \qquad (2) \\
\varepsilon_{33} &= \varepsilon_{rr} = -\frac{\nu}{1-\nu}(\varepsilon_{11} + \varepsilon_{22}) \qquad \varepsilon_{13} = \varepsilon_{23} = 0, \\
\varepsilon_{12} &= \varepsilon_{\theta\vartheta} = \frac{2l}{g \cdot r \cdot \sin\theta} \cdot \frac{\delta^2 W}{\delta\theta\,\delta\vartheta} - \frac{2l\cos\theta}{g \cdot r \cdot \sin^2\theta} \frac{\delta W}{\delta\vartheta},
\end{aligned}$$

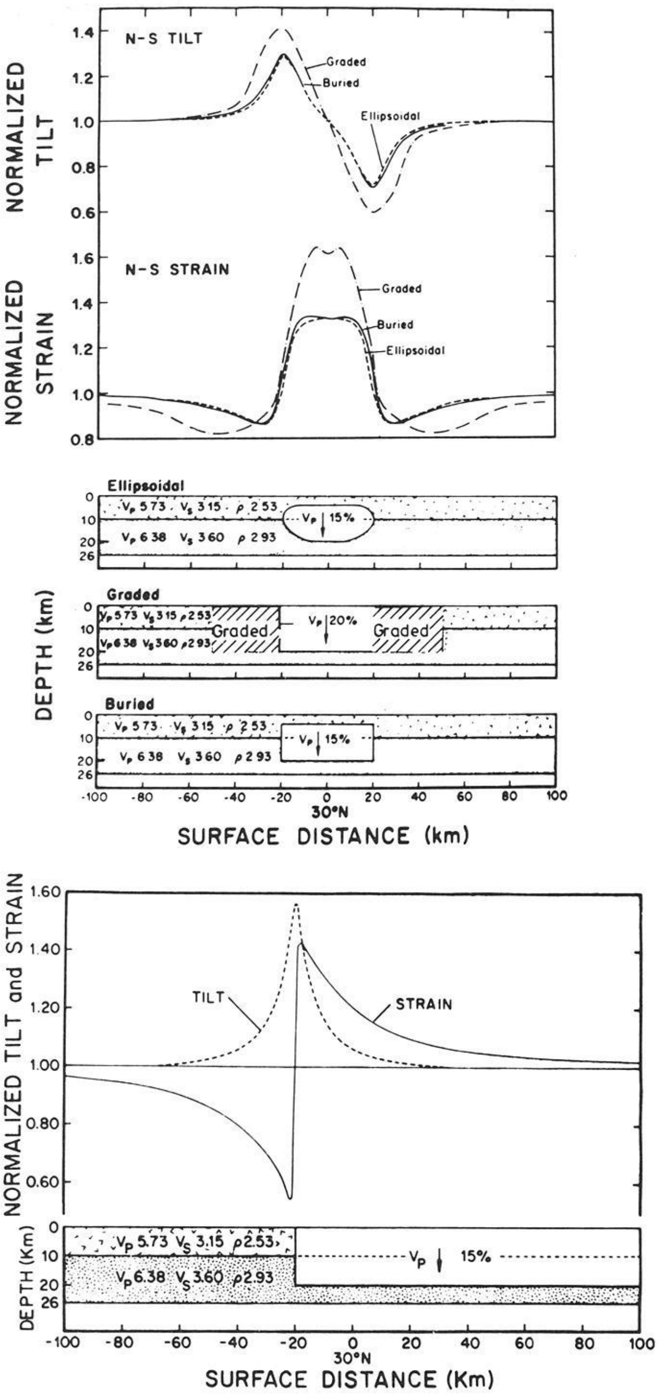

Fig. 1. Finite-element results for tidal modifications due to dilatant regions (after Beaumont and Berger, 1974)

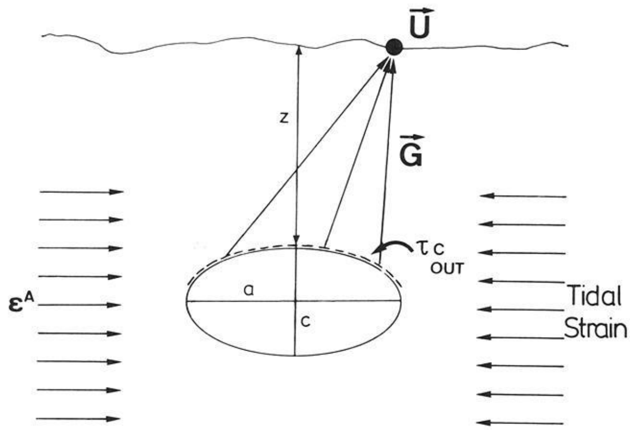

Fig. 2. The ellipsoidal dilatant inclusion: basics for the calculation of tidal modifications

where v = Poisson's ratio for the Earth's crust, h, k, l = Love's numbers (Melchior, 1978) and g = gravity acceleration.

Eshelby (1957) gives the relationship connecting an external strain field ε^A [here the tidal strain field given by Eq. (2)] to the strain field caused by an ellipsoidal inclusion embedded in an otherwise homogeneous, isotropic and unlimited matrix

$$\varepsilon_{ij}^c(\text{out}) = Q_{ijmn}\,\varepsilon_{mn}^A \tag{3}$$

[Eq. 6 of Beavan (1978), who gives a very good explanation of Eshelby's method].

Due to the restrictions of Eshelby's method to the unlimited matrix, one cannot obtain the DZE tidal strain on the Earth's surface by Eq. (3). We use formula (3) merely to calculate the strain field just outside the inclusion and the corresponding stress field is given by Hooke's law.

$$\tau_{ij}^c(\text{out}) = \lambda\,\varepsilon_{ij}^c(\text{out}) + 2\mu\,\varepsilon_{ij}^c(\text{out}). \tag{4}$$

These stresses can be regarded as being caused by a layer of forces surrounding the inclusion.

To transfer these forces to the Earth's surface we make use of Galerkin vectors given by Mindlin and Cheng (1950). By means of these vectors the displacement field at the free surface of a body due to double forces with and without moment acting inside can be calculated by their partial derivatives as shown in Eq. 7. The layer of forces surrounding the inclusion is taken as a layer of double forces without moment for the normal components of $\tau_{ij}^c(\text{out})$ or of double forces with moment for its shear components.

So the Galerkin vector $\mathbf{g}(x, y)$ for the observation point at the Earth's surface corresponding to the forces at a point (x', y') on the surface of the inclusion is given by the multiplication between the Galerkin vectors for double forces with and without moment and the corresponding components of the stress tensor $\tau_{ij}^c(\text{out})$ for the point (x', y')

$$\mathbf{g}(x, y) = \sum_{i=1}^{3}\sum_{j=1}^{3} \mathbf{g}_{ij}(x, y, x', y') \cdot \tau_{ij}^c(\text{out})(x', y'). \tag{5}$$

So the total Galerkin vector $\mathbf{G}(x, y)$ for the observation point is given by the integral over the surface of the inclusion:

$$\mathbf{G}(x, y) = \int_S \mathbf{g}(x, y)\, ds. \tag{6}$$

The displacement vector of the observation point is obtained from the components of the Galerkin vector using

$$u(x, y) = \frac{1}{2\mu}\left[2(1-\nu)\Delta G_x - \frac{\delta}{\delta x}\operatorname{div}\mathbf{G}(x, y)\right]$$
$$v(x, y) = \frac{1}{2\mu}\left[2(1-\nu)\Delta G_y - \frac{\delta}{\delta y}\operatorname{div}\mathbf{G}(x, y)\right] \quad (7)$$
$$w(x, y) = \frac{1}{2\mu}\left[2(1-\nu)\Delta G_z - \frac{\delta}{\delta z}\operatorname{div}\mathbf{G}(x, y)\right]$$

where G_x, G_y, G_z are the components of $\mathbf{G}(x, y)$ (Mindlin and Cheng, 1950).

This leads to strain and tilt given by

$$\mathrm{STRAIN_{NS}}(x, y) = \frac{\delta U(x, y)}{\delta x},$$
$$\mathrm{STRAIN_{EW}}(x, y) = \frac{\delta V(x, y)}{\delta y},$$
$$\mathrm{TILT_{NS}}(x, y) = \frac{\delta W(x, y)}{\delta x}, \quad (8)$$
$$\mathrm{TILT_{EW}}(x, y) = \frac{\delta W(x, y)}{\delta y}.$$

In the following, the DZE tidal effects are normalized by the regular tidal tilt and strain for a radially symmetric Earth without inclusion. This is done using Eq. (2) for the tidal strain.

The tidal tilt is calculated (Melchior, 1978) by

$$\mathrm{TILT_{NS}}(\text{regular}) = (1+k-h)\frac{1}{r\cdot g}\frac{\delta W}{\delta\theta},$$
$$\mathrm{TILT_{EW}}(\text{regular}) = (1+k-h)\frac{1}{r\cdot g\cdot\sin\theta}\frac{\delta W}{\delta\vartheta}. \quad (9)$$

In summary, the calculation of the DZE tidal effects is done in three steps (Fig. 2):

– the stress field τ^c_{out} just outside an ellipsoidal inclusion is calculated using the formalism given by Eshelby (1957)

– the Galerkin vector $\mathbf{G}(x, y)$ belonging to this stress field is calculated for an observation point at the Earth's surface using the formulas of Mindlin and Cheng (1950)

– the displacement vector $\mathbf{u}$ and the resulting tilts and strains are obtained by partial derivatives of the Galerkin vector $\mathbf{G}$.

The relationship between the external strain field ε^A and the strain field $\varepsilon^c_{\text{out}}$ due to the inclusion is given by the matrix $\mathbf{Q}$, Eq. (3). It can be seen that each component of $\varepsilon^c_{\text{out}}$ is obtained by a linear combination of all components of ε^A. The normal components of ε^A are proportional to the cosine of the doubled hour angle ϑ, while the shear components are proportional to the sine of this angle. So the DZE tidal effect is a combination of a sine part and a cosine part, while all regular N-S and E-W tidal effects are proportional to the cosine of the doubled hour angle. Consequently, the phase shift between an arbitrary DZE tidal effect and its corresponding undisturbed tidal effect is given by the arctangent of the ratio of its sine and cosine parts.

The calculated normalized DZE tidal effects are shown along a traverse crossing the buried ellipsoidal inclusion (Fig. 3). For comparison, the geometric and elastic properties of matrix and inclusion, as well as the geographical latitude of the inclusion, are the same as those chosen by Beaumont and Berger (1974) for their model (Fig. 1) with the exception of a thinner overburden (2 km instead of 4 km), a reduced shear modulus inside the inclusion (to obtain a phase shift as explained later) and a semi-axis in the y-direction equal to that in the x-direction.

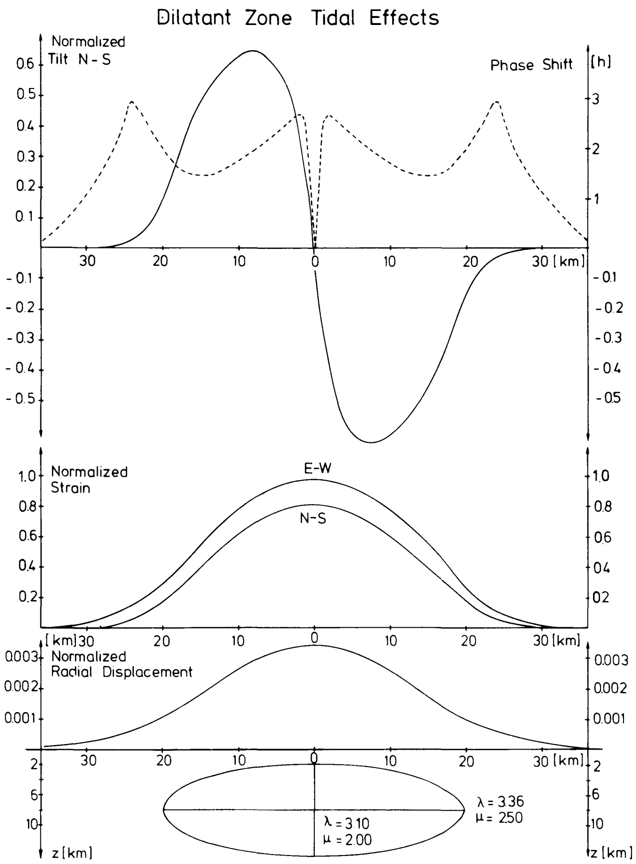

Fig. 3. DZE tidal effect along a N-S profile at 30° N crossing a buried ellipsoidal inclusion

The amplitude of the DZE effects thus obtained are in accordance with those obtained by Beaumont and Berger (1974). The radial displacement along the traverse shows that an additional uplift occurs due to the inclusion, leading to the calculated DZE tilts and strains. Both components of the DZE strain field reach the maximum amplitude just above the centre of the inclusion, the amplitudes being similar to the amplitudes of the regular tidal strain. The maximum DZE N-S tilt is reached at about one-third of the distance from the centre of the inclusion to its edge and the amplitude is about 60 % of the regular tilt amplitude. The maximum phase shift is reached over the edge of the inclusion. The phase shift over the centre of the inclusion is not defined because of zero values for both sine and cosine parts. As these effects are shown along a traverse above the large semi-axis of the inclusion, the DZE E-W tilt along this path is zero. The influences of elastic and geometric parameters of the inclusion on the DZE tidal effects are shown in Figs. 4 and 5. Figure 4 shows the DZE tidal effects caused by a

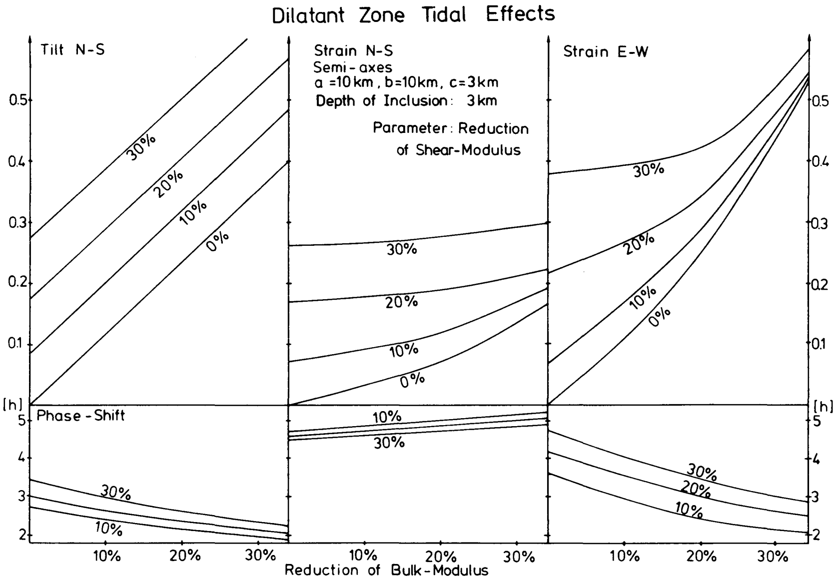

Fig. 4. DZE tidal effects as a function of the reduction of elastic moduli inside the inclusion. Calculations are made for surface points where maximum DZE tidal effects are reached, different for tilt and strain. For N-S tilt this is near the edge of the inclusion, while for both strain components it is above the centre of the inclusion

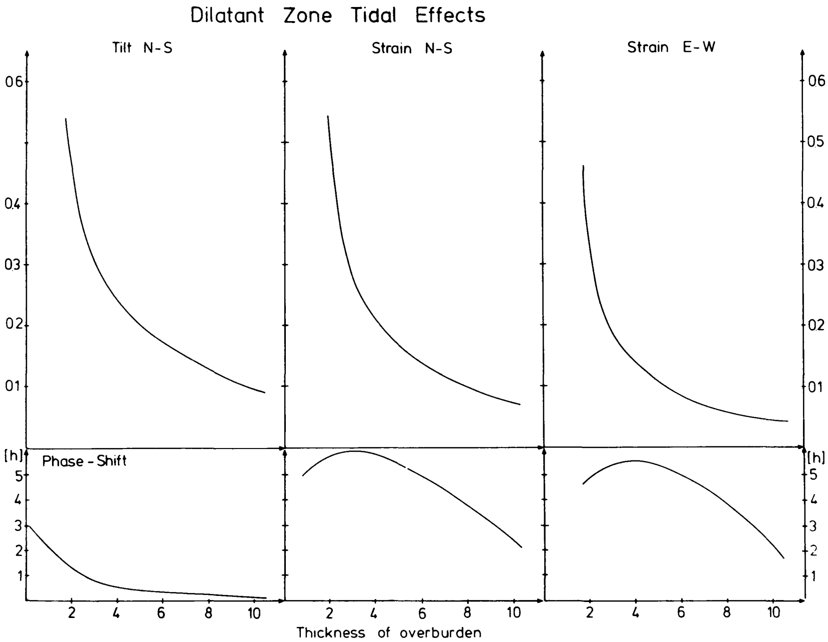

Fig. 5. DZE tidal effects as a function of the thickness of the overburden

reduction of bulk modulus and shear modulus of up to 30% inside the inclusion. Both tilt and strain amplitudes increase with increasing difference of bulk and shear modulus between inside and outside the inclusion.

The phase shifts of N-S tilt and E-W strain reach maximum values when the shear modulus inside the inclusion is reduced while the bulk modulus remains constant, which means that the Poisson's ratio inside the inclusion is higher than outside. The phase behaviour of the N-S strain is contrary to the phase behaviour of the E-W strain and the N-S tilt, which means the phase shift is increasing with a reducing Poisson's ratio inside the inclusion. Since the amplitude of the sine component of all DZE tidal effects is governed by the difference of the shear moduli of matrix and inclusion (as a consequence of Eshelby's matrix Q), no phase delay is obtained for vanishing shear moduli difference.

Among the geometrical parameters, the most crucial one is the thickness of the overburden (i.e. depth of the inclusion) as shown in Fig. 5. All components of the DZE tidal effects show a sharp increase of amplitude for decreasing depth of the inclusion. The limiting case of zero depth can not be calculated because of the restrictions in the formalism given by Eshelby (1957).

Application to earthquake prediction

Prior to an earthquake the physical properties of the crustal material in a certain region near to the future focal area undergo characteristic changes (Scholz et al., 1973). The most striking effect is the decrease in the

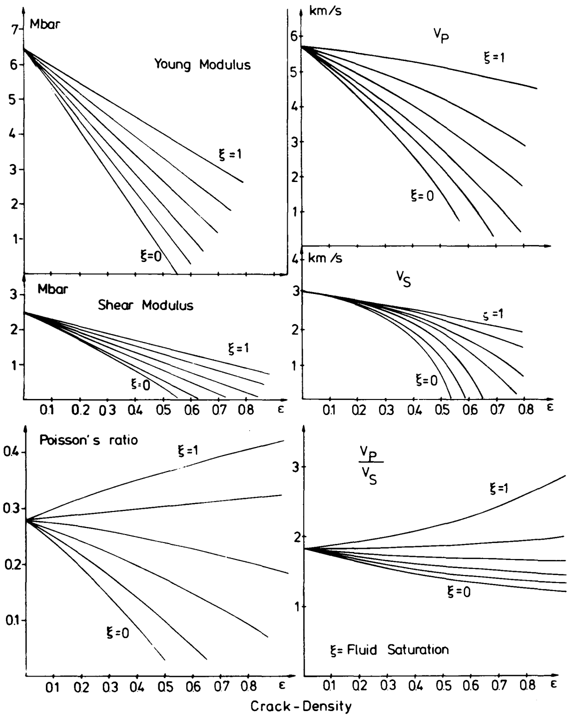

Fig. 6. Elastic moduli and seismic velocities for cracked solids as a function of crack density and pore-fluid saturation (after O'Connell and Budiansky, 1974

v_p/v_s ratio of about 10% followed by a return to normal just prior to the earthquake. An explanation for this behaviour is given by the dilatancy-diffusion theory (Nur, 1972). During the stress-accumulation phase, prior to the earthquake, microcracks in the rock open in a direction parallel to the major compression axis. As a result of this crack-volume increase the pore space becomes undersaturated; this reduces the velocity ratio v_p/v_s and increases the frictional strength of the rock. After a time depending on the extent of the newly created dilatant region, water diffuses into the opened cracks and the velocity ratio recovers. The increased pore pressure decreases the frictional strength of the rock and failure occurs (Gowd and Rummel, 1977).

The change in seismic velocities prior to an earthquake reflects a change in the elastic properties of the crustal material in the vicinity of the future focal region. The influence of cracks and pore-water saturation on the elastic parameters of rocks can be estimated by formulas of O'Connell and Budiansky (1974). Its general behaviour is shown in Fig. 6. Elastic constants are decreasing with increasing crack density ε, a parameter combining the number of cracks per unit volume and the average length of the cracks. This decrease is steeper for dry cracks than for saturated cracks. In the case when more than 70% of cracks are saturated with pore-fluid, Poisson's ratio and the v_p/v_s velocity ratio increases with increasing crack density.

Anderson and Whitcomb (1975) have proposed that the radius L of the dilatant area is connected to the magnitude of the subsequent earthquake by the empirical relation

$$\log L = 0.26M + 0.46.$$

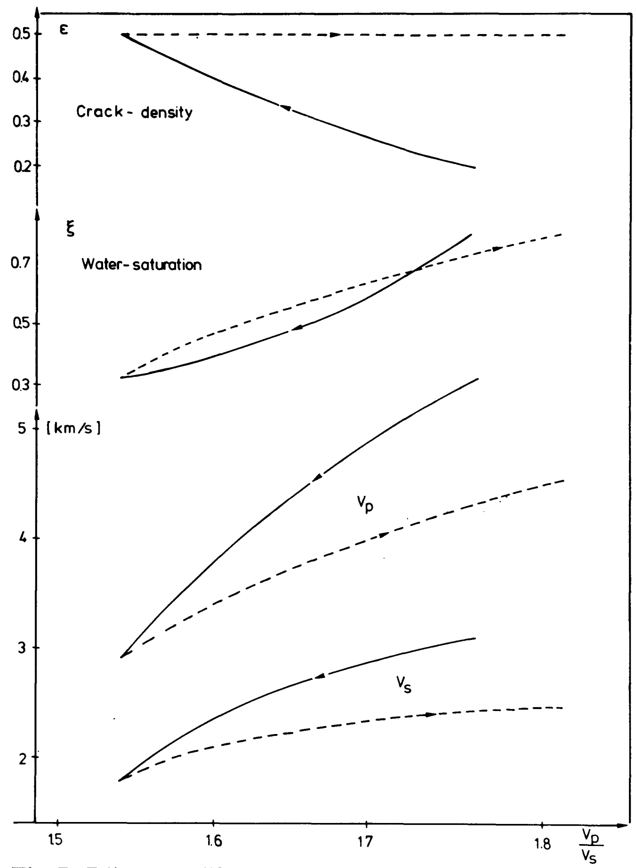

Fig. 7. Dilatancy-diffusion diagram and resulting seismic velocities (after O'Connell and Budiansky, 1974. The *continuous line* indicates the crack-opening phase, while the *dashed line* refers to the water-saturation phase

This suggests that the dilatant region may be much more extended than the aftershock zone. In connection with the reduced elastic parameters within the dilatant zone this leads to the expectation that such a dilatant region can produce DZE tidal effects as described in the previous section. To demonstrate this idea further, consider a dilatancy-diffusion cycle (Fig. 7) containing decreasing pore-space saturation due to the increase in crack density and the subsequent recovery of saturation while crack density remains constant. The model might be an oversimplification of the mechanism but it enables us to estimate the magnitude of the DZE effects to be expected. The resulting seismic velocities inside the dilatant region, obtained from the method given by O'Connell and Budiansky (1974), are shown in Fig. 7. They are used to determine the elastic parameters inside an inclusion having the same geometrical properties as the one used in Fig. 4. The resulting DEZ tidal effects are shown in Fig. 8. The maximum amplitudes for DZE tilt and strain are obtained at the lowest v_p/v_s ratio, while the maximum phase delay occurs during the phase of velocity recovery. Both DZE tilt and strains are in the order of magnitude of the undisturbed body tide effects.

The next step is the calculation of the DZE tidal effects for an actual earthquake. The 1973 $M=2.6$ Blue Mountain Lake earthquake was chosen for this test because exact estimations of velocity anomalies as well as dimensions of the dilatant region have been pub-

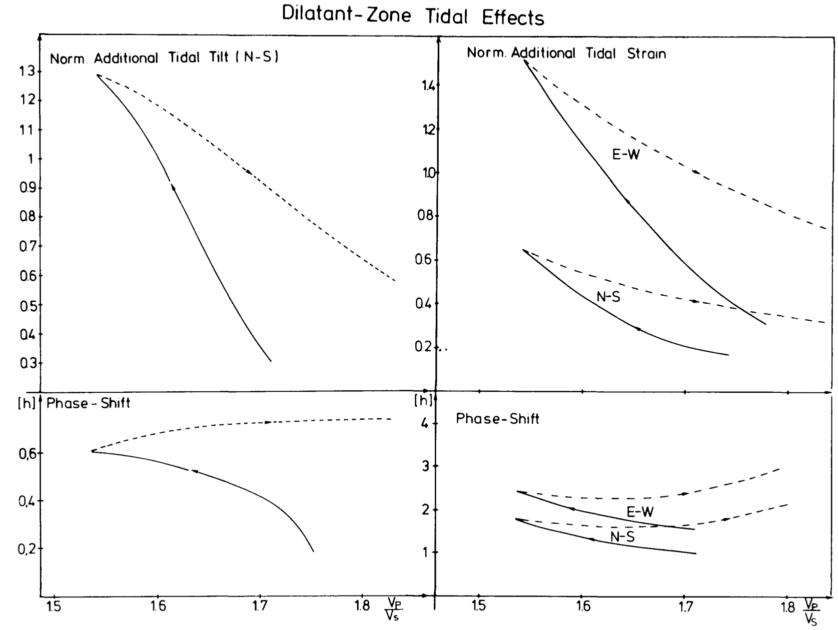

Fig. 8. DZE tidal effects resulting from a dilatancy-diffusion cycle

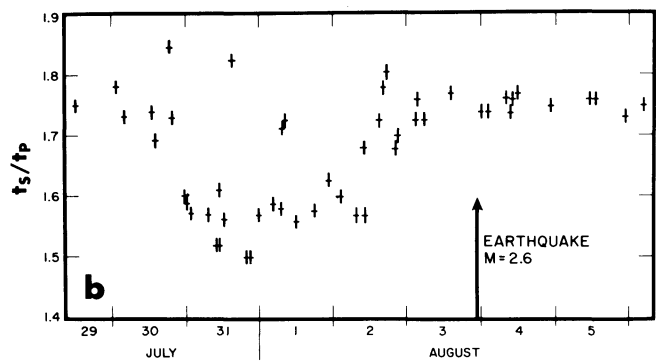

Fig. 9. Observed v_p/v_s ratios prior to the $M=2.6$ Blue Mountain Lake Earthquake (after Aggarwal et al., 1975)

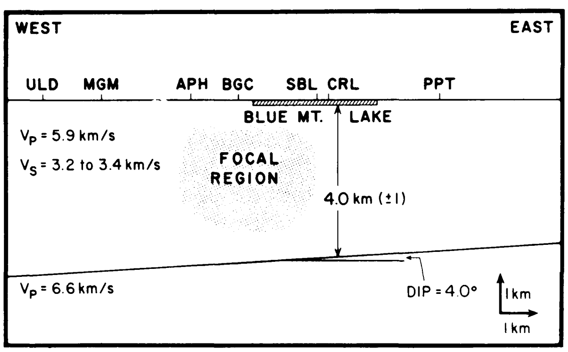

Fig. 10. Dimensions of the dilatant region of the $M=2.6$ Blue Mountain Lake Earthquake (after Aggarwal et al., 1975

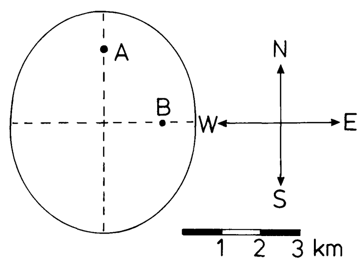

Fig. 11. Position sketch of the imaginary tidal observation points A and B above the semi-axes of the ellipsoid approximating the dilatant zone of the Blue Mountain Lake earthquake

lished (Aggarwal et al., 1975). The decrease of the v_p/v_s ratio (Fig. 9) and its subsequent increase are evaluated in terms of crack density and saturation degree to obtain the elastic parameters inside the dilatant region. The dilatant region (Fig. 10) is approximated by an ellipsoidal inclusion with semi-axes of 2 km (horizontal E-W), 2.5 km (horizontal N-S) and 1.3 km (vertical); the thickness of the overburden is 1 km. In order to calculate both E-W and N-S components of DZE tidal tilt, two imaginary observation points are chosen situated above the horizontal semi-axes of the inclusion (Fig. 11). The results are shown in Fig. 12; the ordinate refers to the time scale in Fig. 9, i.e. day no. 1 refers to July 30.

The amplitude of all DZE tidal effects is high because of the thin overburden of the dilatant region. The amplitude reaches its maximum while the v_p/v_s ratio is at a minimum; for all DZE tidal effects, the phase delay is increasing up to 3–4 h just prior to the earthquake.

Conclusions

A model is presented to account for the influence of a buried dilatant inclusion on Earth tide tilts and strains. This model is based on Eshelby's solution of the strain field due to an ellipsoidal inclusion and, because of its 3-dimensional character, not only the magnitude but also the phase of the induced tilt and strain can be calculated. For the limiting case of a very elongated ellipsoid (corresponding to the 2-dimensional case) results on the magnitude similar to those of Beaumont and Berger (1974) and Molodensky (1983) were obtained.

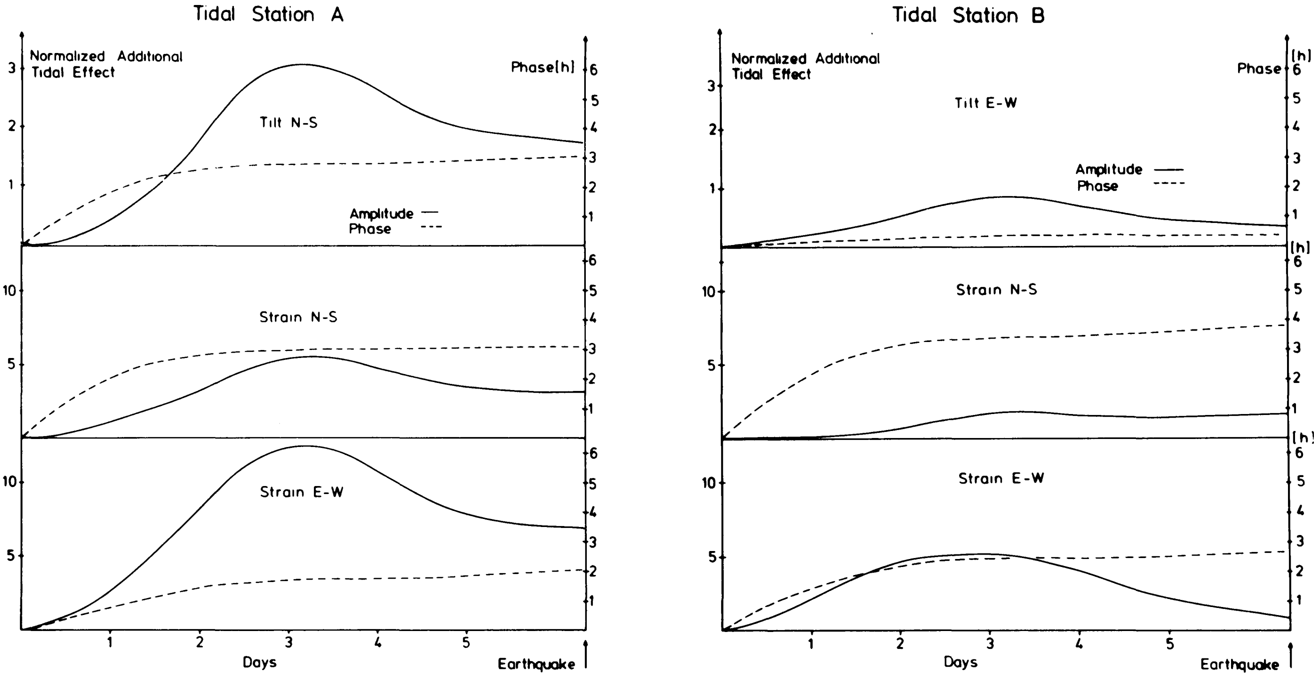

Fig. 12 A, B. DZE tidal effects prior to the Blue Mountain Lake earthquake using the v_p/v_s ratio shown in Fig. 9

The magnitude is determined by the geometrical properties of the inclusion and the contrast between the elastic parameters in the inclusion and in the surrounding material. A very crucial parameter is the thickness of the overburden. No significant tidal effect due to the inclusion can be expected for a thickness of overburden of more than 10 km. Two restrictions of the presented model must be taken into account. First, the dilatant zone must be of ellipsoidal shape, so no statement about the influence of irregularities in the shape of the inclusion can be made. Also, the effect of a gradual transition from the dilatant region to the surrounding crustal material, instead of a sharp boundary, cannot be modelled. Beaumont and Berger (1974) showed that this would slightly reduce the DZE tilt and strain.

Second, the rheological parameters of the dilatant region were obtained by the observed reduction of seismic velocities which were interpreted as changes only in the linear elastic parameters λ and μ. However, the most striking effect of dilatancy is the nonlinear inelastic increase in rock volume prior to failure. These influences, including possible anisotropy of elasticity, could not be modelled. Due to this objection, the model presented here should be taken as an order of magnitude estimation of the amplitude and phase of tidal effects due to a buried dilatant inclusion in the Earth's crust. The pre-seismic tidal modifications calculated under the above-stated assumptions are, nevertheless, extremely high (up to several hundred percent in amplitude and several hours in phase), and they consequently suggest that tidal observations are a viable tool for the prediction of shallow earthquakes.

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