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Seismic signal velocity in absorbing media

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Abstract. One-dimensional synthetic-seismogram calculations are reported for homogeneous media with a quality factor which depends on frequency according to a power law. The purpose is to find out with which signal velocity, i.e. onset velocity, the weakly dispersed body-wave pulses propagate. Three types of seismic waves are simulated: (1) long-period teleseismic P and S waves, (2) short-period regional S waves, (3) P_g waves in explosion seismology. In all cases the signal velocity is close to the group velocity at the dominant period of the source pulse. Phase velocity describes the onsets less accurately. The difference between group and phase velocity becomes significant when the dissipation time t^* exceeds the dominant period. This is the case when the propagation distance exceeds Q times the dominant wavelength.

Key words: Absorption – Group velocity – Phase velocity – Signal velocity

Introduction

Absorption of seismic waves is connected with weak dispersion. Even in a homogeneous medium plane-wave propagation (or far-field propagation from a point source) is characterized by pulse-form changes, and it is not clear with which velocity a special feature of the pulse, e.g. the onset, an extremum or a zero crossing propagates. For seismological applications the most interesting question is with which velocity the wave onset travels. Our definition of the wave onset refers to the time at which about 1% of the maximum amplitude is reached; this definition is plausible for noise-free and unclipped seismic body-wave pulses, as the seismogram examples given later show. The onset velocity will be called signal velocity, in agreement with the definition by Brillouin (1960). The specific question that we will attempt to answer is the following: is signal velocity closer to phase velocity or to group velocity at a representative frequency of the wave, or can neither of these velocities be preferred? The usual stationary-phase argument by which *group* velocity is favoured works only in cases with well-developed dispersion, where a particular frequency can be assigned to a particular time, but not in cases of weak dispersion, where a wave pulse experiences broadening and amplitude decay, but remains a pulse.

The literature on the dispersion of seismic body waves offers quite different answers to the question posed. For instance, Futterman (1962, p. 5281) states: "For sufficiently small absorption group velocity can be identified with the velocity with which a measured signal propagates ...". Similarly, Brennan and Stacey (1977) remark on group velocity: "This is the observed velocity in the case of a body wave ...". A less definitive statement is from Carpenter (1981, p. 422f): "Neither the phase velocity ... nor the group velocity ... are appropriate ...". Similarly, Minster (1980, p. 180) writes: "This implies that the average signal velocity from the origin decreases as time and distance increase". In another paper this author calculates travel times in an earth model from *phase* velocities (Minster, 1978). Finally, Strick (1970), Kjartansson (1979) and Chin (1980) in papers on one-dimensional waves in viscoelastic media discuss travel-time aspects, partly at great length, without mentioning group velocity at all. This brief survey of some of the literature illustrates that the question of seismic signal velocity in absorbing media needs more study. It is shown below that accurate numerical calculations can settle the problem for absorptive conditions as they exist in the earth.

Before presenting these results we discuss briefly a theoretical argument, and its limitations, in favour of group velocity as the essential velocity for body-wave travel times. For the sake of simplicity we consider a plane wave in a weakly dispersive *elastic* medium. The Fourier representation of this wave is

$$S(z,t)=\frac{1}{\pi}\,\mathrm{Re}\int_0^\infty \bar{S}(0,\omega)\,e^{j(\omega t-kz)}\,d\omega, \tag{1}$$

where $\bar{S}(0,\omega)$ is the wave spectrum at $z=0$. In the case of weak dispersion, the real wavenumber $k(\omega)$ can be approximated linearly:

$$\begin{aligned} k(\omega) &= k(\omega_0)+k'(\omega_0)(\omega-\omega_0)\\ &= \left(\frac{1}{c_0}-\frac{1}{U_0}\right)\omega_0+\frac{\omega}{U_0}. \end{aligned} \tag{2}$$

Here $c_0=c(\omega_0)=\omega_0/k(\omega_0)$ is the phase velocity and $U_0 = U(\omega_0)=1/k'(\omega_0)$ the group velocity at the frequency ω_0 which is taken in the frequency band of the input. Inserting

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Eq. (2) into Eq. (1) yields the following exact result:

$$S(z,t)=\cos\chi\cdot S\left(0,t-\frac{z}{U_0}\right)-\sin\chi\cdot S_H\left(0,t-\frac{z}{U_0}\right), \quad (3)$$

$$\chi=\left(\frac{1}{c_0}-\frac{1}{U_0}\right)\omega_0 z.$$

$S_H(0,t)$ is the Hilbert transform of the waveform $S(0,t)$ at $z=0$. The dispersed wave at coordinate z, therefore, is a linear combination of the source pulse and its Hilbert transform, delayed by the *group* travel time z/U_0. This result can be considered as giving some support to the statement that it is group velocity, and not phase velocity, which determines onset times. Brillouin (1960), in a discussion of mainly electromagnetic waves which includes contributions by Sommerfeld and others from the early twentieth century, presents analytical calculations with a similar result. However, the weights $\cos\chi$ and $\sin\chi$ in Eq. (3) depend on z such that, as expected, the pulse form changes with z and hence also the character of the onset. Moreover, it is not immediately evident which frequency ω_0 should be chosen; it is plausible, but not necessary, to take the dominant frequency. More definitive conclusions can only be derived from numerical experiments.

Synthetic seismograms

We have investigated one-dimensional wave propagation in a homogeneous medium whose anelastic quality factor has power-law dependence on frequency:

$$Q(\omega)=Q(\omega_r)\left(\frac{\omega}{\omega_r}\right)^{\gamma}, \quad 0\leqslant\gamma<1. \quad (4)$$

Here ω_r is a reference frequency at which Q is known. Equation (4) represents cases with practical relevance for seismology: $\gamma=0$ is the often-used case of constant Q, and positive γ values have been suggested on the basis of seismological observations (Anderson and Minster, 1979; Ulug and Berckhemer, 1984; Schmidt, 1986) and laboratory experiments (Berckhemer et al., 1982; Kampfmann, 1984).

The velocity dispersion of a medium with Q according to Eq. (4) has been investigated, among others, by Brennan (1980) and Müller (1983). The phase velocity $c(\omega)$, the group velocity $U(\omega)$ and the dissipation operator $A(\omega,z)$ are $(0<\gamma<1)$:

$$c(\omega)=c(\omega_r)\left\{1+\frac{1}{2Q(\omega_r)}\cot\frac{\gamma\pi}{2}\left[1-\left(\frac{\omega_r}{\omega}\right)^{\gamma}\right]\right\}, \quad (5)$$

$$U(\omega)=c(\omega)/\left(1-\frac{\gamma}{2Q(\omega)}\cot\frac{\gamma\pi}{2}\right), \quad (6)$$

$$A(\omega,z)=\exp\left\{-\frac{\omega z}{2c(\omega_r)Q(\omega_r)}\left(\left(\frac{\omega_r}{\omega}\right)^{\gamma}-j\cot\frac{\gamma\pi}{2}\left[1-\left(\frac{\omega_r}{\omega}\right)^{\gamma}\right]\right)\right\}. \quad (7)$$

The results for frequency-independent $Q(\gamma=0)$ are:

$$c(\omega)=c(\omega_r)\left(1+\frac{1}{\pi Q}\ln\frac{\omega}{\omega_r}\right), \quad (8)$$

Table 1. Parameters of the wave-propagation cases studied

Case		T (s)	$\omega_r/2\pi$ (Hz)	$c(\omega_r)$ (km/s)	$Q(\omega_r)$	γ	z (km)
Long-period	P	10	1	10	600	0, 0.3	0–10,000
	S			6	300		
Short-period	S	1	1	6	300	0.3	0–4,000
Explosion seismology (P_g phase)		0.1	1	5	200	0.3	0–100

$$U(\omega)=c(\omega)/\left(1-\frac{1}{\pi Q}\right), \quad (9)$$

$$A(\omega,z)=\exp\left\{-\frac{\omega z}{2c(\omega_r)Q}\left(1-\frac{2j}{\pi}\ln\frac{\omega}{\omega_r}\right)\right\}. \quad (10)$$

The dispersion implied by Eqs. (5) and (6) or by Eqs. (8) and (9) is anomalous $(U>c)$ and inverse (U increases with frequency). A few numerical results for c and U can be found in Müller (1983).

The meaning of the dissipation operators (7) and (10) is the following. The wave spectrum at the coordinate z in an anelastic medium is obtained from the elastic spectrum, corresponding to propagation with the known phase velocity $c(\omega_r)$ at the reference frequency, through multiplication by $A(\omega,z)$:

$$\bar{S}(z,\omega)=\bar{S}(0,\omega)\,e^{-j\omega z/c(\omega_r)}A(\omega,z). \quad (11)$$

Equations (5)–(11) are practically exact for frequencies for which dissipation is slight $[Q(\omega)\gg 1]$; at least several frequency decades above and below ω_r can be considered, if $Q(\omega_r)\gg 1$ and γ is not too close to 1. In the cases studied below, the condition $Q(\omega)\gg 1$ is always satisfied.

Equation (11) has been used to calculate synthetic seismograms $S(z,t)$ for the source pulse $S(0,t)=\sin(2\pi t/T)-0.5\sin(4\pi t/T)$, normalized to unit amplitude $(0\leqq t\leqq T)$. T is the dominant period of the input, and the spectrum is effectively confined to frequencies from 0 to $3/T$. Three different cases have been studied: the first simulates propagation of long-period body waves to teleseismic distances, the second corresponds to the propagation of short-period S waves to regional distances and the third simulates the P_g wave of explosion seismology. Table 1 summarizes the parameters of the three cases. The numerical calculation of synthetic seismograms in the frequency domain via Eq. (11) poses no problems; in order to avoid time-domain aliasing, which can be severe for larger propagation distances, we used the standard method of complex frequencies. The seismograms calculated in this way can be considered as exact and thus well suited for inferences on travel times and signal velocity.

The synthetic record sections in Figs. 1–3 have been reduced with the velocity $c(\omega_r)$. Each seismogram is normalized by its maximum amplitude. This is the adequate procedure for a theoretical study of signal velocity; it agrees also with the routine practice of seismograms display in explosion seismology. The tick marks in the seismograms correspond to the time when the amplitude has reached 1% of the maximum amplitude. This is approximately the time where an observer would place the onset in a noise-free and unclipped trace. The straight travel-time lines in the

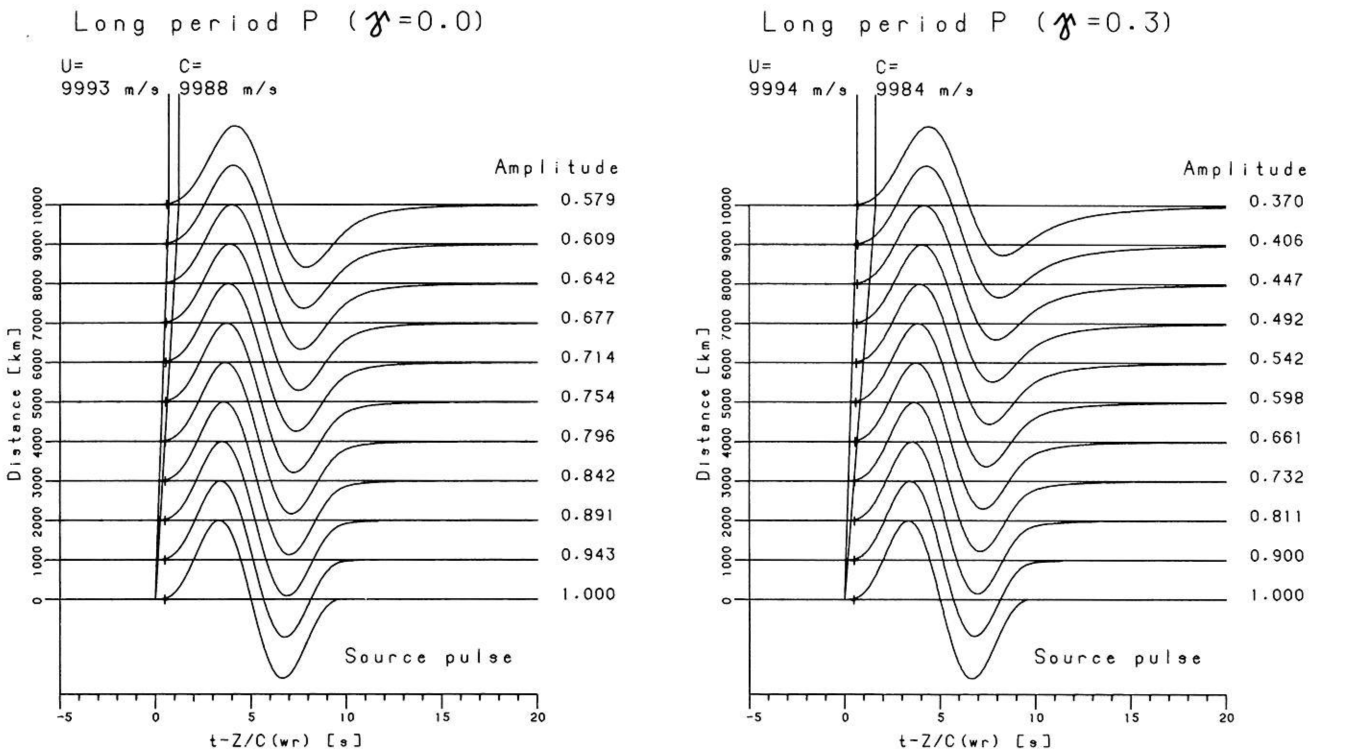

Fig. 1. Synthetic seismograms for the long-period *P*-wave case of Table 1; $\gamma=0$ (*left*) and $\gamma=0.3$ (*right*). The seismograms are normalized, with peak amplitudes given at the end of the traces. The *tick marks* corresponds to 1% of the peak amplitudes. The travel-time lines follow from group velocity U and phase velocity c, respectively, at the dominant frequency of the source pulse

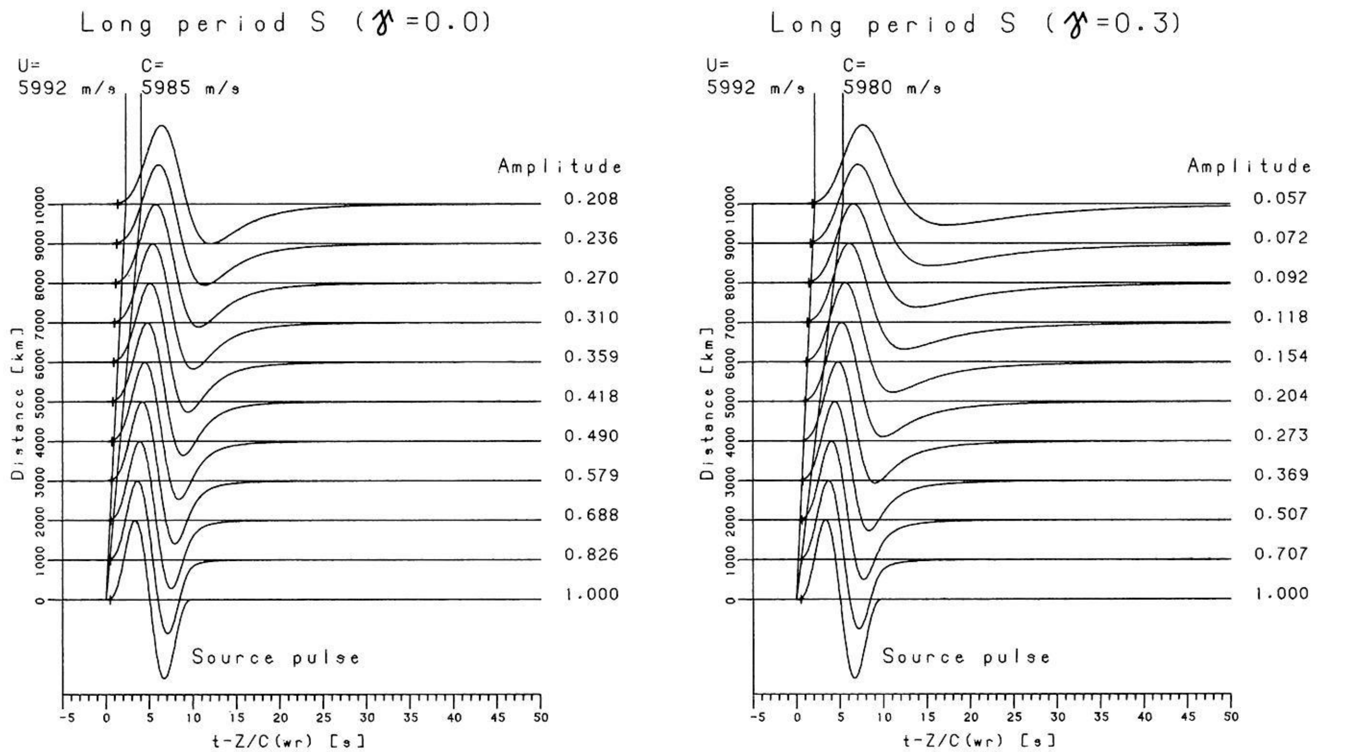

Fig. 2. The same as Fig. 1 for the long-period *S*-wave case

record sections start at $z=0$ and correspond to the phase and the group velocity, respectively, taken at the dominant frequency $1/T$ of the source pulse. It is obvious that in all sections group velocity explains the onsets at larger distances considerably better than phase velocity. Only at short distances is phase velocity sometimes better suited, in particular in the case of Fig. 1. However, at these distances the difference between group-velocity travel time and phase-velocity travel time is so small, compared with the pulse duration, that it is of no interest to discriminate the two.

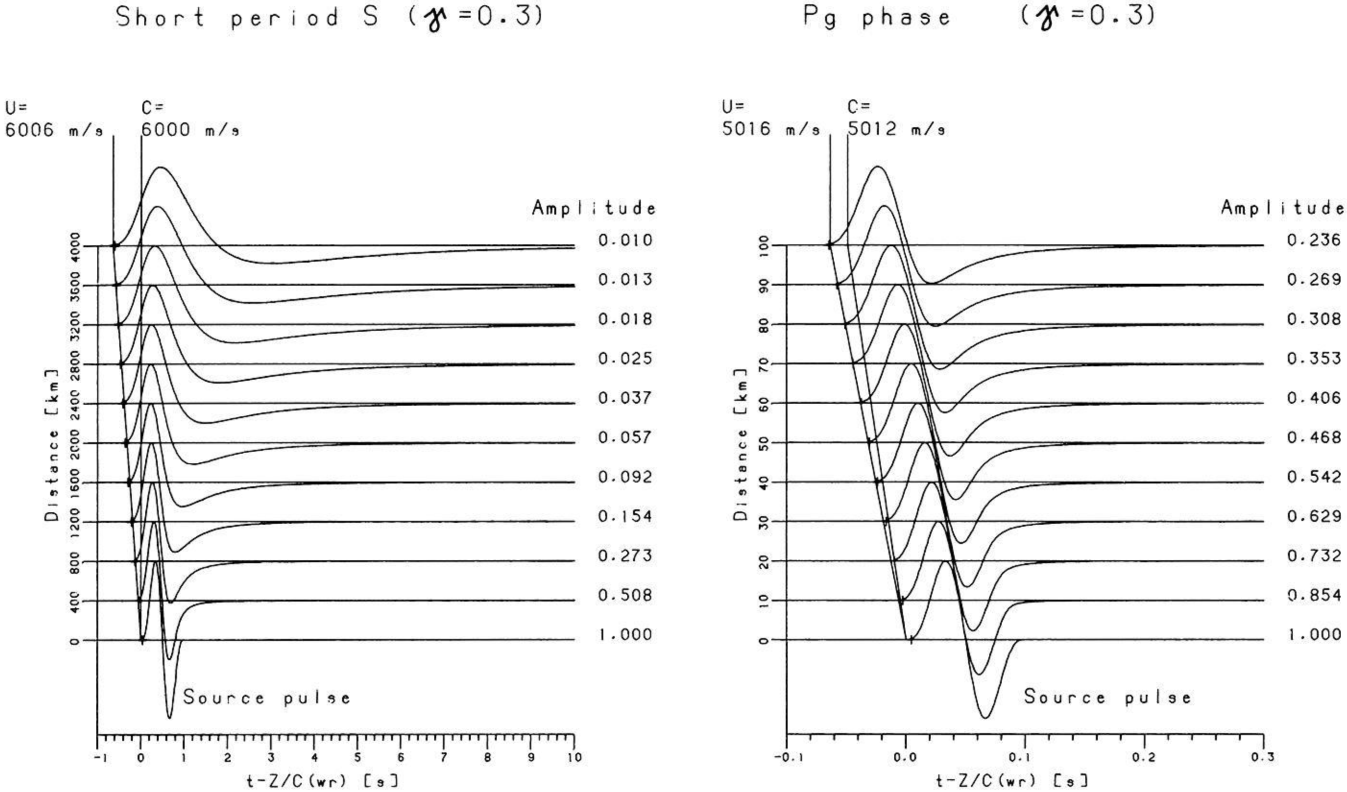

Fig. 3. The same as Fig. 1 for the short-period *S*-wave case (*left*) and the explosion-seismology case (*right*)

Thus, the conclusion from all three cases investigated is that the signal velocity is well represented by group velocity at the dominant frequency of the wave at the source; phase velocity usually is less accurate in explaining onset times.

Discussion and conclusions

Our results are numerical and apply, strictly speaking, only in the particular case studied, characterized by the Q law (4) and the source pulse chosen above. However, these results certainly are typical and apply also in other cases where the source spectrum is bandlimited with a relatively well-defined dominant frequency, and where significant absorption exists in this frequency band.

It is of interest to have a rule of thumb, by which one can approximately decide when it is advisible to work with group rather than phase velocity. Group velocity should be chosen when the travel-time difference $z/c(\omega)-z/U(\omega)$ at the dominant frequency $\omega=2\pi/T$ exceeds a significant fraction of the dominant period T, say T/π. Inserting Eqs. (5) and (6) gives a simple condition in terms of the dissipation time $t^*=z/[c(\omega)\,Q(\omega)]$ at the dominant frequency:

$$\frac{t^*}{T}>\frac{1}{\dfrac{\gamma\pi}{2}\cot\dfrac{\gamma\pi}{2}}\,. \tag{12}$$

The function on the right side of Eq. (12) is 1 for $\gamma\to 0$, 1.05 for $\gamma=0.25$ and 1.27 for $\gamma=0.5$. Thus, for a relatively broad range of γ values the right side of Eq. (12) is about 1. The conclusion, then, is that the difference between group and phase velocity becomes significant if the dissipation time t^* exceeds the dominant wave period T. In this case, one should use the group velocity for travel-time calculations. The condition $t^*>T$ can be translated into the condition $z>\lambda Q$ for the propagation distance z, where λ is the dominant wavelength and Q corresponds to the dominant frequency.

A conclusion from our investigations, with perhaps some practical relevance, concerns the Preliminary Reference Earth Model (PREM) of Dziewonski and Anderson (1981). In the construction of PREM, velocity dispersion was assumed to follow the laws for frequency-independent Q, but body-wave travel times were calculated with the phase velocity (8) rather than the group velocity (9). According to our results, the joint inversion of free-oscillation periods and travel times should, in principle, incorporate the distinction of phase and group velocity. In practice, this distinction may be marginally relevant for S waves. Should PREM be revised in the future, it may be reasonable to calculate body-wave travel times with group velocity.

Finally, we want to emphasize the approximate nature of the statement that signal velocity agrees with group velocity at about the dominant frequency of the source pulse. This agreement is valid when the shift of the dominant frequency with increasing propagation distance is *moderate*, such that the dominant frequency of the source pulse is present also at large distances. If this frequency is completely lost, either due to strong absorption or due to extremely long propagation distances, group velocity at this frequency also loses its significance. Signal velocity then becomes distance-dependent, and the group velocity at about the *local* dominant frequency may well be a good approximation. We have not tested this idea numerically since in the earth extremely strong absorption, connected with amplitude decay by orders of magnitude, normally does not occur.

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