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The relation between Born inversion and standard migration schemes

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Abstract. Born inversion represents a novel approach in reflection seismic data processing. Omission of multiple scattering effects and relying on a known reference velocity allows for true amplitude mapping of subsurface structures. Born inversion is closely related to classical migration concepts in terms of its provisions as well as its results. Consequent application of high-frequency approximations, generally accepted in reflection seismology, leads to simple relations between Born inversion, frequency-wavenumber migration and migration based on a Kirchhoff summation technique. The formulas are derived for the simple case of 3-D zero-offset geometry with a constant reference velocity. Modifications of Born inversion theory are presented if the particle velocity is measured instead of the pressure and if the influence of the free surface is incorporated.

Key words: Reflection seismology – Migration – Born inversion

Introduction

The seismic experiment aims at the extraction of information concerning a certain volume of space from measurements of physical quantities like pressure, particle displacement or velocity on parts of a boundary enclosing the volume. Refraction seismology derives velocity functions averaged over several wavelengths in space; the targets of reflection experiments are structural features of the subsurface. Actually, both notions, smoothed velocities and structural information, in general, refer to the same physical parameter – compressional velocity – but mean different spectral components. Velocity functions inverted from refraction data represent the low-wavenumber portion of the true velocity distribution, whereas reflection data allow for the inversion of the reflective properties of the structure caused by the high-wavenumber part of the true velocity function, see Claerbout (1985, pp. 46).

An intermediate result of reflection seismic data processing is the common-midpoint stacked section, that can be considered as an approximation of a zero-offset section defined by coincident source-receiver geometry. These sections represent seismic images of the earth, of the position and the strength of subsurface reflectors. It is, however, a distorted image. Dipping reflectors, for instance, appear at positions shifted away from the true ones. In addition, hyperbolically arranged diffraction onsets mark their end points. The reconstruction of the true image of the subsurface structure is the ultimate goal of seismic migration. Originally the efforts concentrated on the reconstruction of geometric features, e.g. location and extend of reflectors, and thus required only geometric methods (Haagedorn, 1954). During the last 15 years migration was based on the acoustic wave equation and various approaches of its solution like ray theory, finite-difference methods, the Kirchhoff approach, and operations in the frequency-wavenumber domain. In general, a wave field recorded at the surface is downward continued and then subjected to an imaging condition. An overview on migration methods can be found in Robinson (1983) and Claerbout (1985).

Standard migration procedures model a zero-offset section with reflectors exploding simultaneously at a certain time, say $t=0$, and waves travelling upward in a medium with half the true velocity. The migration process downward continues the wave field and defines the (band-limited) reflectivity as its value at the imaging time $t=0$.

If one accepts the accoustic wave equation as appropriate for the description of reflection seismological phenomena, its exploitation in migration or inversion procedures could possibly lead to the reconstruction, not only of the proper position of reflectors but also to amplitudes that measure and express the reflection strength adequately and thus deliver an additional parameter for the interpretation. Any method aiming at this goal is called true amplitude migration. Born inversion, introduced, worked out and refined in recent years, belongs to this category. Its development is associated with the names of Bleistein, Cohen, Hagin and other workers, e.g. see Bleistein and Cohen (1982) or Cohen et al. (1986).

As Born inversion represents a rather new approach to the inverse problem of reflection seismology, one would like to know its similarities with and differences to standard migration procedures in terms of the basic physical and mathematical models used, and in terms of assumptions and approximations employed. This paper attempts to address this question and, besides the contribution of Bleistein and co-authors, uses the results of Cheng and Coen (1984) who also discuss the relationship between Born inversion and migration of common-midpoint stacked data.

In order to keep the analysis as simple as possible the following considerations are restricted to a constant reference velocity and to zero-offset geometry. An inversion aims at the true-amplitude recovery of the reflectivity of the earth. Therefore the basis of the inversion, the forward model, e.g. the wave equation and Born's approximation, must be critically discussed in as far as it properly describes the physics of the seismic experiment. Another question refers to the measured quantity used for inversion, e.g. pressure or particle velocity, and the quantity desired as output, e.g. velocity perturbation or reflectivity. For an inversion it can be demonstrated that two essentially equivalent approaches, in spatial and in Fourier domain, are conceivable.

The basic equation

This section presents a detailed derivation of Born inversion for a case that reveals the main features of the theory but is simple enough to avoid cumbersome mathematics. The basic equation involved is the Helmholtz equation in three-dimensional (3-D) space.

$$\left[\nabla^2+\frac{\omega^2}{v^2(\mathbf{x})}\right]p(\mathbf{x},\mathbf{x}_s,\omega)=-\delta(\mathbf{x}-\mathbf{x}_s)$$
$$\mathbf{x}_s=(x_s,y_s,0)^T,\qquad \mathbf{x}=(x,y,z)^T. \tag{1}$$

It describes the behaviour of the pressure in a liquid medium with no or negligible density variations. In order to utilize Eq. (1) for reflection seismology the medium is assumed to consist of an upper halfspace with constant velocity continuously connected with a lower halfspace containing an arbitrary velocity distribution. The inhomogeneous part of the Helmholtz equation (1) represents a point source in the observational plane defined by $z=0$ with a spike-like, broadband source signal. The source radiates the pressure in all directions. Whereas no reflection can be expected from the upper halfspace, the variable velocity structure of the lower halfspace reflects a certain amount of energy that can be recorded in the observational plane.

The Helmholtz equation (1) and the concept of velocity structure do not ideally model the conditions of the reflection seismic experiment. Clearly the elastic equations should be employed instead of the acoustic equation. The upper halfspace should be simulated with a zero velocity, introducing free surface effects. The band limitation of the seismic source requests an additional frequency function in the inhomogeneous part of Eq. (1). Despite these shortcomings the Helmholtz equation (1) is generally considered as an appropriate mathematical tool in reflection seismology mainly because steep angle propagation and steep angle reflections prevent any relevant conversion of the source-generated compressional waves to shear waves; the medium behaves as if it were liquid. Consequently, the Helmholtz or the wave equation became the basis for most modern migration schemes. Nevertheless, the deficiencies of the equation as a model for reflection seismic experiments should be kept in mind, in particular whenever true amplitudes are concerned.

Delineation of 3-D structural features of the subsurface requires a 3-D seismic survey with sources and receives distributed in the whole observational plane $z=0$. The seismograms used as input data for the Born inversion procedure discussed in this paper are zero-offset traces. In theory these seismograms could be recorded with coincident source-receiver geometry. In practice, such a measurement is of little use because shot-generated noise completely overrides the signals reflected from the subsurface. However, common-midpoint stacked data can be viewed as an approximation

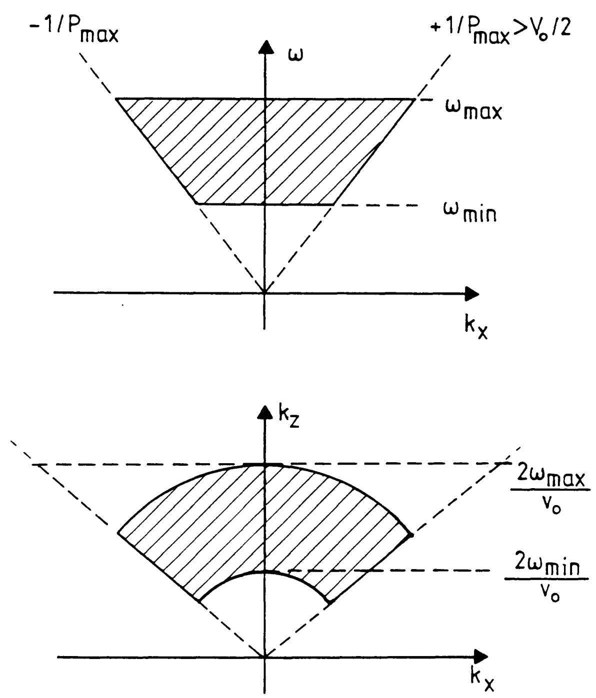

Fig. 1. Sketch of temporal and spatial band limitation for reflection data of a zero-offset section with reference velocity v_0. The *upper part* shows that available information is restricted to a frequency band (ω_{min}, ω_{max}) and a ray parameter window ($\pm p_{max}$). *Lower part* indicates the resulting map to the (k_x, k_z) plane utilizing the dispersion relation

of a zero-offset seismogram containing only the reflective response of the medium.

The ultimate goal of an inversion procedure based on Eq. (1) for seismic data is the derivation of the spatial velocity distribution. Born inversion does not fully accomplish this task. The velocity field is composed of a smooth, long-wavelength, low-wavenumber part and another one with short-wavelength oscillations. The low-wavenumber part, commonly referred to as the reference velocity, controls travel times and must be known as input for the inversion. Large-wavenumber constituents of the velocity field are responsible for the reflectivity of the medium. Only these components of the velocity field form the target of Born inversion. The following derivation assumes a constant reference velocity in order to keep the analysis as simple as possible. A treatment of arbitrary reference velocities can be found in Cohen et al. (1986).

The Helmholtz equation (1) is written for a source without any band limitation. In reality, however, field data as well as stacked sections are band-limited in frequency and wavenumber or ray-parameter content. Physical limitations of the source, the propagation properties of the medium and the receivers reduce the frequency content of seismic data to typically 10–100 Hz. In zero-offset sections the maximum ray parameter observable is $2/v_0$, twice the inverse of the reference velocity. Again the theoretical value often exceeds the true maximum encountered. One reason is the wavenumber (ray parameter) filtering property of arrays of sources and receivers commonly used as field techniques. In addition, zero-offset rays are often close to vertical.

The data consisting of a seismic trace for each receiver position (x_g, y_g) at the surface form a 3-D space in the observational variables (x_g, y_g, t). The limited information attainable with reflection seismic measurements can be demonstrated in the corresponding Fourier space (k_x, k_y, ω), where the data are confined to a slice of a cone defined by the frequency band and the maximum ray parameter. Figure 1 (upper panel) shows a cut through the cone in the $k_y=0$ plane. The data are used for the derivation of 3-D structural information, e.g. the velocity perturbation $a(x, y, z)$ or the reflectivity $c(x, y, z)$. The mapping function that defines the corresponding domain in Fourier space (k_x, k_y, k_z) that can be reconstructed from the data is the dispersion relation:

$$k_z^2 = \frac{4\omega^2}{v_0^2} - k_x^2 - k_y^2.$$

The lower panel of Fig. 1 shows this domain as a 2-D sketch. The 3-D situation is achieved by rotation around the k_z axis. Most calculations that follow in this paper are based on Eq. (1) and thus ignore the band-limited character of the data. This is acceptable because the relation between structural parameters and data is linear and the derivation would thus not be different. All one has to keep in mind is that the final integral operations are applied to band-limited functions.

Born approximation

An appropriate formulation of the forward problem serves as a starting point for the inversion. In the time domain in the acoustic wave equation governs the propagation of pressure fluctuations (p), excited by a broad-band source, located in the observational plane defined by $z=0$ at $(x_s, y_s, 0)$, in a liquid medium with constant density and arbitrary spatial velocity distribution. In the frequency domain the Helmholtz equation (1) replaces the acoustic wave equation; its unique solution is guaranteed by Sommerfeld's radiation conditions. The z-axis points downward, into the earth. A constant velocity characterizes the upper halfspace (negative z-axis). The velocity field in the lower halfspace is partitioned into a reference value $\bar{v}$ and a dimensionless perturbation of v so that

$$\frac{1}{v^2(\mathbf{x})} = \frac{1}{\bar{v}^2(\mathbf{x})}[1 + a(\mathbf{x})]. \tag{2}$$

A corresponding decomposition of the pressure into

$$p = p_I + p_s$$

defines a pressure p_I whose propagation is controlled by $\bar{v}$:

$$\left[\nabla^2 + \frac{\omega^2}{\bar{v}^2(\mathbf{x})}\right] p_I(\mathbf{x}, \mathbf{x}_s, \omega) = -\delta(\mathbf{x} - \mathbf{x}_s). \tag{3}$$

The solution of this equation yields Green's function $g(\mathbf{x}, \mathbf{x}_s, \omega)$ for the reference velocity field.

If Eq. (2) and (3) are inserted into Eq. (1), an inhomogeneous Helmholtz equation results:

$$\left[\nabla^2 + \frac{\omega^2}{\bar{v}^2(\mathbf{x})}\right] p_s(\mathbf{x}, \mathbf{x}_s, \omega) = -\frac{\omega^2}{\bar{v}^2(\mathbf{x})} a(\mathbf{x})[p_I(\mathbf{x}, \mathbf{x}_s, \omega) + p_s(\mathbf{x}, \mathbf{x}_s, \omega)]. \tag{4}$$

As the right-hand side acts as source term, Green's function allows for the integral representation of the solution as a spatial convolution of the inhomogeneous term and Green's function

$$p_s(\mathbf{x}_g, \mathbf{x}_s, \omega) = \omega^2 \int \frac{a(\mathbf{x})}{\bar{v}^2(\mathbf{x})} [p_I(\mathbf{x}, \mathbf{x}_s, \omega) + p_s(\mathbf{x}, \mathbf{x}_s, \omega)]\, g(\mathbf{x}_g, \mathbf{x}, \omega)\, d^3\mathbf{x} \tag{5}$$

with the geophone location

$$\mathbf{x}_g = (x_g, y_g, z_g)^T.$$

Note that no approximations have been used so far.

Equations (3) and (5) represent just another view of the Helmholtz equation (1). The solution for the total pressure is approached by first finding Green's function for the reference velocity field, and then solving an integral equation that connects the residual pressure p_s with the previously evaluated Green's function and the scattering potential

$$\omega^2 \frac{a(\mathbf{x})}{\bar{v}^2(\mathbf{x})}.$$

As it stands the reformulation of the original Helmholtz equation is of little use because neither does it lend itself to exact solution techniques of the forward problem nor does it prove useful for an inversion. It serves, however, as the basis for the introduction of physical assumptions that allow for the Born approximation. Suppose that the reference velocity $\bar{v}$ is known a priori, e.g. from refraction surveys or NMO analysis, then the scattering potential or the velocity perturbation remains the target of inversion. The solution of Eq. (3) is Green's function which incorporates all kinds of multiples, upward and downward travelling waves if $\bar{v}$ varies in space. For Born's approximation, a solution containing only downward-propagating waves is sufficient. Such a solution results, for example, from splitting the wave equation into portions that yield separated upward- and downward-directed waves or from high-frequency approximations like asymptotic ray theory. Then p_I can be viewed as the wavefield generated by a source, travelling directly to the velocity perturbation, interacting with it and thus generating the scattered field p_s. Weak scattering can be defined by

$$|p_s| \ll |p_I|$$

which allows us to neglect p_s in the integrand of Eq. (5). This assumption is called Born's (first) approximation. Taking into account the particular geometry used in reflection seismology, and the circumstance that scattering comes exclusively from the lower halfspace, leads to

$$p_s(\mathbf{x}_g, \mathbf{x}_s, \omega) = \omega^2 \int \frac{a(\mathbf{x})}{\bar{v}^2(\mathbf{x})} g(\mathbf{x}, \mathbf{x}_s, \omega)\, g(\mathbf{x}_g, \mathbf{x}, \omega)\, d^3\mathbf{x} \tag{6}$$

where, in addition, symbol p_I is replaced by Green's function, the solution of Eq. (3). Equation (6) relates the pressure measured in the observational plane $z=0$, through a linear Fredholm integral equation, to the unknown velocity perturbation $a(x, y, z)$. As the incident wavefield remains unaffected by the velocity perturbation, transmission losses or refractive effects caused by $a(x, y, z)$ are not considered.

Omitting the scattered wavefield in the integral in Eq. (5) implies that only the incident waves interact with the velocity perturbation, never the scattered waves. Consequently, multiple effects are not incorporated and again transmission and refraction phenomena of the upward-propagating scattered wave caused by $a(x, y, z)$ are neglected. The rather general Eq. (6) and its utilization for the inverse theory can be found in Cohen et al. (1986). In this paper the simple case of a constant reference velocity v_0,

$$\frac{1}{v^2(\mathbf{x})}=\begin{cases}\frac{1}{v_0^2} & z\leqq 0\\ \frac{1}{v_0^2}[1+a(\mathbf{x})] & z>0,\end{cases}$$

is considered. With Green's function for this particular case

$$g(\mathbf{x},\xi,\omega)=\frac{e^{j\frac{\omega}{v_0}|\mathbf{x}-\xi|}}{4\pi|\mathbf{x}-\xi|}$$

and coincident source and receiver coordinates, Eq. (6) becomes

$$p_s(\mathbf{x}_g,\omega)=\left(\frac{\omega}{4\pi v_0}\right)^2\int a(\mathbf{x})\frac{1}{r^2}e^{j\frac{2\omega}{v_0}r}d^3\mathbf{x} \tag{7}$$

with

$$r=[(x-x_g)^2+(y-y_g)^2+z^2]^{\frac{1}{2}}.$$

With a guess of the reference velocity v_0 and broad-band observations in the plane $z=0$, one aims at the inversion of the linear integral equation (7) to derive the velocity perturbation. Two points should be mentioned before the inversion itself is tackled. Firstly, p standing for pressure in (7) specifies the measured quantity only in marine experiments and not in land-based reflection seismology where the vertical particle velocity replaces p. Secondly, the velocity perturbation does not represent an appropriate quantity measurable with and invertible from reflection data. These items will be discussed in detail later.

The inversion

An inversion of Eq. (7) can pursue two approaches. The first method utilizes spatial Fourier transforms and tries to find a simple, readily invertible relation between data and velocity perturbation in Fourier space. The second approach designs an approximate inverse operator without transforming the data.

Transform method

An attempt to Fourier transform Eq. (7) with respect to x_g any y_g encounters the problem that an analytic expression for the transform of

$$\frac{1}{r^2}e^{j\frac{2\omega}{v_0}r}$$

is not available. If Eq. (7) could be manipulated in a way that only the inverse of r instead of the inverse square of r appears, then Weyl's integral for the plane-wave decomposition of a point source allows the evaluation of the correct Fourier transform:

$$\frac{e^{j\frac{2\omega}{v_0}r}}{4\pi r}=\frac{1}{8\pi^2}\iint_{-\infty}^{+\infty}\frac{j}{v}e^{j[k_x(x-x_g)+k_y(y-y_g)+v|z|]}dk_x\,dk_y$$

$$v=\begin{cases}\operatorname{sgn}(\omega)\left(\frac{4\omega^2}{v_0^2}-k_x^2-k_y^2\right)^{\frac{1}{2}}, & \frac{4\omega^2}{v_0^2}\geqq k_x^2+k_y^2\\ j\left(k_x^2+k_y^2-\frac{4\omega^2}{v_0^2}\right)^{\frac{1}{2}}, & \frac{4\omega^2}{v_0^2}<k_x^2+k_y^2.\end{cases} \tag{8}$$

The introduction of modified data

$$q_s=-j\frac{\partial}{\partial\omega}\left(\frac{1}{\omega^2}p_s\right) \tag{9}$$

leads to the desired modification of Eq. (7)

$$q_s(\mathbf{x}_g,\omega)=\frac{1}{2\pi v_0^3}\int a(\mathbf{x})\frac{e^{j\frac{2\omega}{v_0}r}}{4\pi r}d^3\mathbf{x}. \tag{10}$$

The right-hand side of Eq. (10) represents an integral over the product of velocity perturbation and Green's function for a homogeneous space with velocity $v_0/2$. Therefore, the modified data must be a solution of

$$\left(\nabla^2+\frac{4\omega^2}{v_0^2}\right)q_s(\mathbf{x},\omega)=-\frac{a(\mathbf{x})}{2\pi v_0^3} \tag{11}$$

in a plane specified by $z=0$. This equation describes the propagation of a quantity q_s in a space with velocity $v_0/2$, whose sources explode at $t=0$. Its strength and spatial distribution is determined by $a(x, y, z)$. In other words, Eq. (11) represents the mathematical formulation of the exploding reflector model of Loewenthal et al. (1976), the well-known basis for migration of zero-offset sections. Born theory quantifies the exploding reflector model. It reveals the physical meaning of the source strength and the propagating and measured quantity. Whereas the standard view of the exploding reflector model assumes that the source strength is defined by the reflectivity and that the propagating quantity represents the pressure, Born theory tells us, rather, that this model relates velocity perturbation with modified data.

A Fourier transformation of both sides of Eq. (10), utilizing Eq. (8), results in

$$Q_s(k_x,k_y,\omega)=\frac{j}{4\pi v_0^3 v}A(k_x,k_y,-v) \tag{12}$$

where $Q_s(k_x, k_y, \omega)$ is the 2-D Fourier transform of $q_s(x_g, y_g, \omega)$ and

$$A(\mathbf{k})=\int a(\mathbf{x})\,e^{-j\mathbf{kx}}d^3\mathbf{x},\qquad \mathbf{k}=(k_x,k_y,k_z)^T. \tag{13}$$

The simple relation between Q_s and A in Fourier space can be considered as solution of the inverse problem

$$A(k_x,k_y,-v)=-4\pi j v_0^3 v\cdot Q_s(k_x,k_y,\omega). \tag{14}$$

In order to express the solution in spatial coordinates $a(x, y, z)$, an inverse Fourier transform must be performed. The inverse of Eq. (13) is

$$a(\mathbf{x})=\frac{1}{(2\pi)^3}\int A(\mathbf{k})\,e^{j\mathbf{kx}}\,d^3\mathbf{k}. \tag{15}$$

As Eq. (14) expresses A in the variables $(k_x, k_y, -v)$ rather than (k_x, k_y, k_z), the variables of integration in Eq. (15) must be changed according to

$$k_x=k_x$$
$$k_y=k_y$$
$$k_z=-\operatorname{sgn}(\omega)\left(\frac{4\omega^2}{v_0^2}-k_x^2-k_y^2\right)^{\frac{1}{2}}. \tag{16}$$

The definition of the vertical component of the wave vector in Eq. (8) suggests that A can be determined for real and complex values. For the inversion, however, the real k_z values are sufficient. The mapping is one-to-one, the full 3-D space in (k_x, k_y, k_z) is projected into a double cone in (k_x, k_y, ω). With the functional determinant

$$\frac{-4\omega}{v_0^2 v}$$

Eq. (15) can be written as

$$a(\mathbf{x})=\frac{1}{2\pi^3 v_0^2}\iiint\frac{\omega}{v}\,A(k_x, k_y, -v)\cdot e^{-jvz}e^{j(k_x x+k_y y)}\,dk_x\,dk_y\,d\omega, \tag{17}$$

where the expression for A as a function of k_x, k_y and $-v$ can be replaced by Q_s using Eq. (14):

$$a(\mathbf{x})=\frac{2v_0}{j\pi^2}\iiint\omega Q_s(k_x, k_y, \omega)\cdot e^{-jvz}e^{j(k_x x+k_y y)}\,dk_x\,dk_y\,d\omega. \tag{18}$$

This equation solves the inverse problem. It relates the velocity perturbation $a(x, y, z)$ to the modified data q_s with a formula strongly reminiscent of results from standard frequency-wavenumber migration, e.g. see Stolt (1978), Gazdag (1978) or Castle (1982). The physical view of migration is, however, quite different because there a wavefield propagating in a medium with velocity $v_0/2$ and recorded at $z=0$ is downward continued and then subjected to an imaging process.

Equation (18) solves the inverse problem correctly without assumptions beyond those inherent in Born's approximation. A simplification of Eq. (18) reintroducing the pressure is feasible if only high frequencies are involved. Again, two conceivable approaches can be pursued. The first one starts with Eq. (18) and relies on partial integration; the second one completely omits the modified data. It relinquishes a correct Fourier transform of Eq. (7) and uses a stationary-phase approximation instead.

Application of the rule of partial integration to the ω-integral in Eq. (18) results in

$$\int_{-\omega_1}^{\omega_1}\omega Q_s\,e^{-jvz}\,d\omega$$
$$=-j\left[\frac{P_s}{\omega}e^{-jvz}\right]_{-\omega_1}^{\omega_1}+j\int_{-\omega_1}^{\omega_1}\frac{P_s}{\omega^2}\left(1-\frac{4jz\omega^2}{v_0^2 v}\right)e^{-jvz}\,d\omega.$$

ω_1 indicates the maximum frequency for a particular wavenumber k. As the data will always be band-limited below this value, the first term vanishes. In the expression appearing in the integral on the right-hand side the first term can be neglected if

$$\frac{z}{\lambda}\gg 1,$$

where λ is the maximum wavelength and in real data the dominant wavelength. Therefore, the inversion formula (18) can be written as

$$a(\mathbf{x})=\frac{8z}{\pi^2 v_0}\iiint\frac{1}{jv}\,P_s(k_x, k_y, \omega)\cdot e^{-jvz}e^{j(k_x x+k_y y)}\,dk_x\,dk_y\,d\omega \tag{19}$$

an expression valid for depths exceeding a few wavelengths, e.g. a few hundred metres at most.

The same result arises if a high-frequency approximation is utilized in the Fourier transform of Eq. (7). As an exact transform is unknown, an approximate transform can be derived with the method of stationary phase. This method is commonly applied to integrals over one variable, e.g. see Bath (1968, p. 44) but can be extended to two (Papoulis, 1968, p. 241) or more dimensions (Bleistein, 1984, p. 283). Application of the technique in two variables results in

$$P_s(k_x, k_y, \omega)=\frac{j\omega}{16\pi v_0}\,\bar{A}(k_x, k_y, -v),$$

where $\bar{A}$ is the 3-D Fourier transform of

$$\bar{a}(\mathbf{x})=\frac{a(\mathbf{x})}{z}.$$

The reconstruction of $\bar{a}(x, y, z)$ from $\bar{A}(k_x, k_y, -v)$ follows the procedure outlined with Eq. (15)–(18) and leads directly to Eq. (19). The latter relation clarifies the connection between $f-k$ migration and Born inversion. The velocity perturbation can be recovered from the pressure by standard $f-k$ migration of the weighted pressure

$$\frac{1}{jv}\,P_s(k_x, k_y, \omega)$$

and subsequent multiplication of the result with z. It should be mentioned that a multiplication of the section with z after migration is not identical with a multiplication of the data with time before migration.

If one prefers a Kirchhoff summation technique as the numerical scheme for the inversion, the pressure in Eq. (19) must be written as the Fourier transform of x_g and y_g. If the order of integration is interchanged, Eq. (19) becomes

$$a(\mathbf{x})=\frac{16z}{\pi v_0}\iint p_s(\mathbf{x}_g, \omega)\cdot\left\{\frac{1}{2\pi}\iint\frac{e^{j[k_x(x-x_g)+k_y(y-y_g)-vz]}}{jv}\,dk_x\,dk_y\right\}d^2\mathbf{x}_g\,d\omega.$$

The expression in brackets gives exactly

$$\frac{1}{r}e^{-j\frac{2\omega}{v_0}r}$$

a result readily established if $-\omega$ is inserted in Eq. (8). Therefore,

$$a(\mathbf{x}) = \frac{16}{\pi v_0} \iint \frac{z}{r} p_s(\mathbf{x}_g, \omega)\, e^{-j\frac{2\omega}{v_0} r}\, d^2\mathbf{x}_g\, d\omega \tag{20}$$

or, after a transform to the time domain,

$$a(\mathbf{x}) = \frac{16}{\pi v_0} \int \frac{z}{r} \tilde{p}_s\left(\mathbf{x}_g, t = \frac{2}{v_0} r\right) d^2\mathbf{x}_g. \tag{21}$$

These Kirchhoff formulas state that the velocity perturbation $a(x, y, z)$ can be recovered by summing the weighted pressure along hyperboloids defined by

$$t = \frac{2}{v_0} [(x - x_g)^2 + (y - y_g)^2 + z^2]^{\frac{1}{2}}.$$

Inversion in space

This method tries to design a procedure for directly inverting Eq. (17) in the space domain. It was established by Beylkin (1985) and applied by Cohen et al. (1986) to 3-D Born inversion with arbitrary reference velocity. Equation (7) contains an integral over the velocity perturbation, an amplitude factor B and a phase factor $\omega \cdot \Phi$:

$$p_s(\mathbf{x}_g, \omega) = \left(\frac{\omega}{4\pi v_0}\right)^2 \int a(\mathbf{x})\, B(\mathbf{x}, \mathbf{x}_g)\, e^{j\omega\Phi(\mathbf{x}, \mathbf{x}_g)}\, d^3\mathbf{x}, \tag{22}$$

$$B(\mathbf{x}, \mathbf{x}_g) = \frac{1}{r^2}, \qquad \Phi(\mathbf{x}, \mathbf{x}_g) = \frac{2}{v_0} r,$$

where $\omega \cdot \Phi$ varies rapidly compared to B if the frequency is high. Assume that the solution of the inverse problem can be written in the following form

$$a(\mathbf{x}) = \iint F(\omega)\, I(\mathbf{x}, \mathbf{x}_g)\, e^{-j\omega\Phi(\mathbf{x}, \mathbf{x}_g)}\, p_s(\mathbf{x}_g, \omega)\, d^2\mathbf{x}_g\, d\omega. \tag{23}$$

This equation is the mathematical expression of the intuitive idea of migration. If the pressure is transformed to the time domain, Eq. (23) becomes

$$a(\mathbf{x}) = f(t) * \iint I(\mathbf{x}, \mathbf{x}_g)\, \tilde{p}_s(\mathbf{x}_g, t = \Phi(\mathbf{x}, \mathbf{x}_g))\, d^2\mathbf{x}_g\, dt,$$

where $f(t)$ and $\tilde{p}_s(\mathbf{x}_g, t)$ correspond to $F(\omega)$ and $p_s(\mathbf{x}_g, \omega)$, and $*$ symbolizes temporal convolution. The formula states that migration consists of a summation through the data field along the hyperboloids, defined by

$$t = \Phi(\mathbf{x}, \mathbf{x}_g).$$

$F(\omega)$ and $I(\mathbf{x}, \mathbf{x}_g)$ are still undefined at this stage, but will be determined later. Clearly Eq. (23) is not strictly correct because the exact Eq. (18) has a different form. Equation (20) which displays a form similar to Eq. (23) can be derived from Eq. (18) only if high-frequency approximations are used. The equals sign in Eq. (23) must be understood in this sense. Inserting Eq. (22) into Eq. (23) results in

$$\iint \left(\frac{\omega}{4\pi v_0}\right)^2 F(\omega)\, B(\xi, \mathbf{x}_g)\, I(\mathbf{x}, \mathbf{x}_g)$$
$$\cdot e^{j\omega[\Phi(\xi, \mathbf{x}_g) - \Phi(\mathbf{x}, \mathbf{x}_g)]}\, d^2\mathbf{x}_g\, d\omega = \delta(\xi - x) \tag{24}$$

and thus determines the choice of the functions $F(\omega)$ and $I(\mathbf{x}, \mathbf{x}_g)$. If the phase fluctuates rapidly enough, the integral has only nonvanishing values if $\mathbf{x}$ and ξ coincide or are at least close together. Then one can expand the phase expression with the variables ξ around the variables $\mathbf{x}$, whereas the amplitude $B(\mathbf{x}, \mathbf{x}_g)$ is taken at $\mathbf{x}$. These qualitative considerations can be viewed as a high-frequency approximation because the condition of a rapidly oscillating phase is valid if the frequency is high enough. Beylkin (1985) established these facts more rigorously using results of the theory of generalized Radon transforms. With

$$\Phi(\xi, \mathbf{x}_g) \approx \Phi(\mathbf{x}, \mathbf{x}_g) + \nabla\Phi \cdot (\xi - \mathbf{x}),$$
$$B(\xi, \mathbf{x}_g) \approx B(\mathbf{x}, \mathbf{x}_g), \tag{25}$$

Eq. (24) becomes

$$\iint \left(\frac{\omega}{4\pi v_0}\right)^2 F(\omega)\, B(\mathbf{x}, \mathbf{x}_g)\, I(\mathbf{x}, \mathbf{x}_g)$$
$$\cdot e^{j\omega \cdot \nabla\Phi \cdot (\xi - \mathbf{x})}\, d^2\mathbf{x}_g\, d\omega = \delta(\xi - \mathbf{x}) \tag{26}$$

with

$$\nabla\Phi \cdot (\xi - \mathbf{x}) = \frac{2\omega}{v_0} \cdot \frac{1}{r} (\mathbf{x} - \mathbf{x}_g) \cdot (\xi - \mathbf{x}).$$

A transform of variables according to

$$k_x = \frac{2\omega}{v_0 r} (x - x_g),$$
$$k_y = \frac{2\omega}{v_0 r} (y - y_g),$$
$$k_z = \frac{2\omega}{v_0 r} z,$$

then leads to

$$\frac{1}{(2\pi)^3} \int \frac{\pi v_0 r}{16 z} F(\omega)\, I(\mathbf{x}, \mathbf{x}_g)\, e^{j\mathbf{k}(\xi - \mathbf{x})}\, d^3\mathbf{k} = \delta(\xi - \mathbf{x}).$$

This identity is guaranteed if

$$F(\omega) = 1,$$
$$I(\mathbf{x}, \mathbf{x}_g) = \frac{16 z}{\pi v_0 r}.$$

Thus the quantities $I(\mathbf{x}, \mathbf{x}_g)$ and $F(\omega)$, unknown when Eq. (23) was established, can be inserted there to obtain exactly Eq. (20), the formula for Kirchhoff summation.

The results derived in this section demonstrate the equivalence of different inversion procedures. An exact inversion with modified data and subsequent application of partial integration and a high-frequency approximation leads to the same $(f-k)$ migration formula for the pressure as if the high-frequency assumption is introduced in the evaluation of the Fourier transform of the equation describing the forward problem. In the case treated here, the formula for $(f-k)$ migration can be transformed exactly to a Kirchhoff migration scheme. The straight way to the Kirchhoff summation is the inversion in space without detour to the Fourier domain. The $(f-k)$ formulation of the inverse problem can be rewritten as a Kirchhoff summation even if different slowly varying weighting factors are associated with the pressure. The appropriate stationary-phase approximation is valid if the object of migration is a few wavelengths beneath the surface.

Input and output data

The previous section summarized principles of different inversion procedures with pressure as the input and velocity perturbation as the output quantity. However, only marine surveys record the pressure, whereas in land-based surveys the geophones are sensitive to the vertical component of the particle velocity. In addition, a theory as close as possible to the experimental situation should incorporate the free surface conditions instead of assuming a homogeneous space with some finite velocity above the reflectivity structure.

As indicated in the introduction of this paper, the velocity perturbation is not really the appropriate inversion quantity requested by reflection seismologists. They usually look for the short-wavelength part of the velocity field, e.g. the reflectivity. This requires additional modifications of the inversion results.

Marine versus continental reflection data

In marine reflection experiments, hydrophone streamers positioned several metres below the sea surface record the pressure of the seismic waves. Actually, not only are upcoming waves recorded but also the interaction between them and the reflection from the interface between water and atmosphere ideally associated with a reflection coefficient of -1. In land-based experiments geophones usually record the vertical component w_s of the particle velocity which is related to the pressure in the frequency domain by

$$\frac{\partial p_s}{\partial z_g} = j\omega \rho_0 w_s. \tag{27}$$

ρ_0 represents the density at the geophone position. In order to carry out the calculations of the previous sections with w_s instead of p_s, a derivative with respect to z_g must be introduced in the Born approximation before z_g is set equal to zero. This derivative affects the amplitude and phase in Eq. (5) and thus leads to a near- and far-field term. For high frequencies or a depth of several wavelengths, the near-field contribution can be neglected and Eq. (7) that serves as the starting point of the Born inversion in the marine case must be replaced by

$$w_s(\mathbf{x}_g, \omega) = \frac{2}{\rho_0 v_0}\left(\frac{\omega}{4\pi v_0}\right)^2 \int a(\mathbf{x}) \frac{z}{r^3} e^{-j\frac{2\omega}{v_0} r} d^3\mathbf{x} \tag{28}$$

for continental environments. In Eq. (28) the sign of w_s is chosen in a way that a positive break is caused by a particle moving upwards in the negative z direction. Inversion provides, as the $(f-k)$ formula,

$$a(\mathbf{x}) = \frac{8\rho_0 z}{\pi v_0} \iiint \frac{\omega}{j v^2} w_s(k_x, k_y, \omega) \cdot e^{-j v z} e^{j(k_x x + k_y y)} dk_x\, dk_y\, d\omega \tag{29}$$

and, as the Kirchhoff summation,

$$a(\mathbf{x}) = \frac{8\rho_0}{\pi} \iint w_s(\mathbf{x}_g, \omega)\, e^{-j\frac{2\omega}{v_0} r} d^2\mathbf{x}_g\, d\omega. \tag{30}$$

Comparison to Eq. (20) shows that the weight factor z/r must be omitted in the inversion if the particle velocity is used as the input quantity instead of the pressure.

The influence of the free surface

The influence of the free surface can be introduced in Born's inversion scheme by an appropriate modification of Green's function. Remember that Eq. (6), which solves the forward problem in the frame of Born's approximation, combines a wave travelling downward from a source at $z=0$ with a wave travelling upwards towards the surface from any point inside the reflecting halfspace. This wave is represented by Green's function for a homogeneous space with a source point inside the lower halfspace. If a free surface exists at $z=0$, the wave reflected from the free surface must be added to form the new Green's function

$$g(\mathbf{x}_g, \mathbf{x}, \omega) = \frac{e^{j\frac{\omega}{v_0} r_1}}{4\pi r_1} - \frac{e^{j\frac{\omega}{v_0} r_2}}{4\pi r_2}, \tag{31}$$

where

$$r_{1,2} = [(x-x_g)^2 + (y-y_g)^2 + (z \mp z_g)^2]^{\frac{1}{2}}.$$

The first term represents the upward-travelling wave, the second one the reflected wave. Both are just connected by a subtraction because the free surface acoustic reflection coefficient is -1, independent of the angle of incidence. Again the marine and continental case must be distinguished. For marine pressure data, expression (31) is exactly zero at $z=0$ because there both terms cancel each other. At a depth below the surface Green's function does not vanish and, if z_g is small compared to the dominant wavelength, can be approximately expressed as

$$g(\mathbf{x}_g, \mathbf{x}, \omega) = -\frac{4 j\omega z_g}{v_0} \cdot \frac{z}{r} \cdot \frac{e^{j\frac{\omega}{v_0} r}}{4\pi r}.$$

For continental data the derivative of Eq. (31) with respect to z_g must be evaluated at $z_g=0$. The net effect of this step is the introduction of the scale factor 2 on the right-hand side of Eq. (6). Therefore, the free surface has no consequence if continental data are processed.

Marine data measured at z_g below the sea surface should be inverted or migrated according to

$$a(\mathbf{x}) = \frac{2z}{\pi v_0 z_g} \iiint \frac{1}{v^2} P_s(k_x, k_y, \omega) \cdot e^{-j v z} e^{j(k_x x + k_y y)} dk_x\, dk_y\, d\omega \tag{32}$$

if a $(f-k)$ program is preferred. The associated Kirchhoff formula is

$$a(\mathbf{x}) = \frac{2}{\pi z_g} \iint \frac{1}{-j\omega} p_s(\mathbf{x}_g, \omega)\, e^{-j\frac{2\omega}{v_0} r} d^2\mathbf{x}_g\, d\omega. \tag{33}$$

Comparison with Eq. (20) shows that the weight function z/r must be omitted and a frequency term should be introduced if the influence of the free surface is taken into account.

Reflectivity versus velocity perturbation

Despite the fact that the velocity always acts as a parameter describing the medium in the wave equation, it depends on the experimental configuration whether it might be the appropriate quantity for an inversion. Reflection seismologists know very well that they cannot get reasonable estimates of the velocity; rather, they map the spatial distribution of the reflective properties of the medium. These properties are dominated by the short-wavelength part of the spectrum of the velocity. Its long-wavelength part plays a role in stacking and migrating the data, but these processing steps are not too sensitive to the accuracy of the velocity estimates, and therefore rough spatial averages or estimates that can be determined by, for example, NMO analysis are sufficient.

The amplitude of a reflection is proportional to the reflection coefficient defined as

$$c(z)=\frac{1}{2v(z)}\frac{dv}{dz} \tag{34}$$

in the case of a 1-D velocity variation, constant density and vertical incidence. With the velocity perturbation, defined by Eq. (2) and $|a|\ll 1$, this expression becomes

$$c(z)=-\frac{1}{4}\frac{da}{dz}. \tag{35}$$

The reflectivity can, therefore, be viewed as mainly determined by the small-wavelength contribution to the Fourier spectrum of the velocity. It can be resolved within the bandwidth of the data, e.g. within an upper limitation but also within a lower limit defined by the smallest available frequency. In principle, the velocity perturbation as well as the reflectivity can be used as the inversion target. The velocity perturbation, however, inherently incorporates a large amount of small-wavenumber components, which cannot be resolved with reflection data, whereas the reflectivity emphasizes the high-wavenumber components according to

$$C(k_z)=-\frac{j}{4}k_z\,A(k_z). \tag{36}$$

Figure 2 illustrates this relationship for a thin layer. The reflectivity has a spectrum more equally distributed in the observational wavenumber range compared to the velocity perturbation. It appears, therefore, desirable to obtain the reflectivity as an inversion result. This concept requires a generalization of the definition of the reflectivity to an arbitrary medium with a 3-D velocity fluctuation. A straightforward extension of Eq. (34) is

$$c(\mathbf{x})=\frac{1}{2v(\mathbf{x})}|\nabla v|\,\mathrm{sgn}\left(\frac{\partial v}{\partial z}\right). \tag{37}$$

The formula can be interpreted in the following way: given a spatial velocity distribution, the reflectivity at a point is determined by finding the direction of maximum variation of the velocity. The directional derivative scaled by the velocity defines the modulus of the reflectivity. If the velocity locally increases (decreases) with depth, a positive (negative) reflectivity results. In terms of the velocity perturbation, Eq. (37) becomes

$$c(\mathbf{x})=-\frac{1}{4}|\nabla a|\,\mathrm{sgn}\left(\frac{\partial a}{\partial z}\right). \tag{38}$$

This last equation relates reflectivity and velocity perturbation in space if the parameters vary in three dimensions.

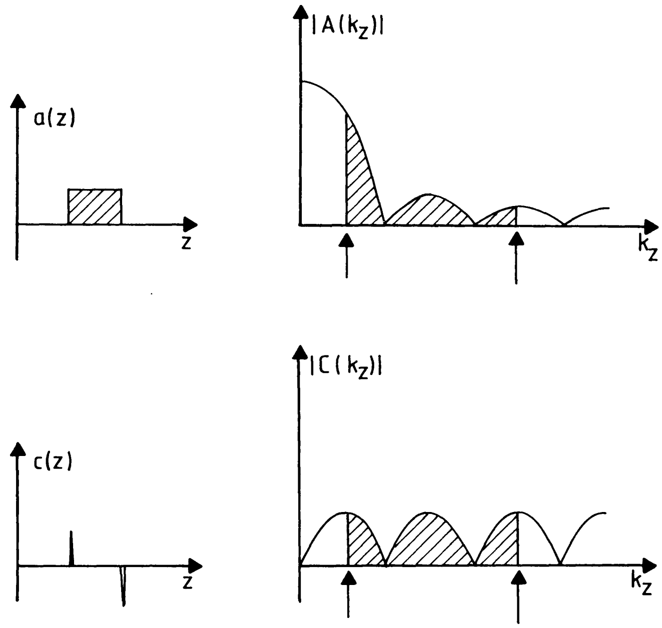

Fig. 2. Demonstration of the distribution of the wavenumber components in the Fourier transform of the velocity perturbation A (*upper part*) and the reflectivity C (*lower part*) for the case of a thin bed. Note that a substantial part of amplitudes for A are concentrated around $k_z=0$ and thus not accessible for band-limited inversion, whereas for C amplitudes are more equally distributed along the k_z axis

For the inversion a relation in the Fourier domain is required, because once available one would simply insert the Fourier transform of the reflectivity for the transformed expression of the velocity perturbation in Eq. (12) and proceed with the inversion along the lines previously described. Whereas for a 1-D medium an exact relation between the transforms of $a(z)$ and $c(z)$ can be given, namely Eq. (36), only an approximate formula can be derived for the general case:

$$C(\mathbf{k})=-\frac{j}{4}|\mathbf{k}|\,\mathrm{sgn}\,(k_z)\cdot A(\mathbf{k}). \tag{39}$$

This relation was proved by Bleistein (1984, p. 289) using stationary-phase approximations. If the medium is composed of domains of constant velocity, its spatial velocity distribution can be written as a sum of characteristic functions

$$v(\mathbf{x})=\sum_i v_i\,\Gamma_i(\mathbf{x}),$$

where $\Gamma_i(\mathbf{x})$ represents a function with unit value inside and vanishing outside a given domain D_i. Inside D_i the velocity is v_i. The different domains are mutually disjoint.

$$\Gamma_i(\mathbf{x})=\begin{cases}1 & \mathbf{x}\in D_i\\ 0 & \mathbf{x}\notin D_i.\end{cases}$$

With

$$a_i=-\frac{2}{v_0}(v_i-v_0),$$

$|a_i|\ll 1$ and Eq. (2), the spatial distribution of the velocity perturbation becomes

$$a(\mathbf{x}) = \sum_i a_i \Gamma_i(\mathbf{x}). \tag{40}$$

The corresponding expression for the reflectivity is

$$c(\mathbf{x}) = \sum_i c_i \gamma_i(\mathbf{x}) \tag{41}$$

with the discrete reflection coefficient

$$c_i = \frac{1}{2v_0}(v_i - v_0)$$

and the singular function $\gamma_i(\mathbf{x})$ which has delta-function-like singularities at the surface where $\Gamma_i(x)$ jumps from zero to unity. Characteristic and singular functions can be viewed as generalisations of the 1-D Heaviside and delta functions. The Fourier transform integrals of $\gamma_i(\mathbf{x})$ and $\Gamma_i(\mathbf{x})$ can be evaluated approximately with the method of stationary phase. Then they are related by

$$\bar{\gamma}(\mathbf{k}) = -j|\mathbf{k}| \operatorname{sgn}(k_z)\, \bar{\Gamma}_i(\mathbf{k}). \tag{42}$$

Equation (39) results if Eqs. (40), (41) and (42) are properly combined. It holds if the medium is composed of areas of constant velocity. Reflectors are located at the interfaces between areas of different velocity. The stationary-phase approximation is reasonable wherever the product of R and k is much greater than unity, where R represents the radius of curvature. If k is associated with a wavelength, then the radius of curvature should exceed the dominant wavelength. Therefore, the approximation fits very well into the resolution capabilities of reflection seismology.

If the reflectivity is used instead of the velocity perturbation, the previously derived results are readily modified. In order to replace $a(x, y, z)$ by $c(x, y, z)$, for instance, in Eq. (14) one inserts

$$k_z = -v$$

into Eq. (39). v is defined in Eq. (8) where only real values are of interest. Then

$$C(k_x, k_y, -v) = \frac{j\omega}{2v_0} A(k_x, k_y, -v).$$

The further steps of the inversion procedure are simply applied to C instead of A. As a result, the data must be multiplied by the frequency factor

$$\frac{j\omega}{2v_0} \tag{43}$$

prior to migration. Then Eq. (18) becomes

$$c(\mathbf{x}) = \frac{1}{\pi^2} \iiint \omega^2 Q_s(k_x, k_y, \omega) \cdot e^{-jvz} e^{j(k_x x + k_y y)} dk_x\, dk_y\, d\omega \tag{44}$$

or Eq. (19) is modified to

$$c(\mathbf{x}) = \frac{4z}{\pi^2 v_0^2} \iiint \frac{\omega}{v} P_s(k_x, k_y, \omega) \cdot e^{-jvz} e^{j(k_x x + k_y y)} dk_x\, dk_y\, d\omega. \tag{45}$$

In the time domain the application of factor (43) corresponds to a differentiation of the seismic traces. If a stacked section of continental data is to be inverted to an image of the spatial distribution of the velocity perturbation, the Kirchhoff formula (30) requires nothing but a straight summation along the diffraction hyperboloids through the section. If an image of the reflectivity is of interest, the data should be differentiated prior to the Kirchhoff summation.

Summary and conclusions

Born inversion represents a new look to the old problem of migration of reflection data. Its similarities with standard wave equation migration schemes originate in the fact that the wave equation is assumed to describe appropriately reflection seismic observations. It uses essentially the same simplifying assumptions about the earth's structure, its division in a slowly varying or even constant background or reference velocity that serves as input to the inversion, and unknown velocity perturbations representing the short-wavelength components of the velocity field causing the reflectivity of the medium. The omission of multiple scattering results in a linear integral equation between data and reflectivity that can be inverted analytically, at least in a stationary-phase sense. The stationary-phase approximations used are acceptable under the conditions generally faced in reflection seismology. The object of inversion is several wavelengths away from the receiver and a curvature is only properly mapped if its radius is larger than a wavelength.

Under these conditions it can be shown that Born inversion applied to a stacked section with constant reference velocity is essentially equivalent to frequency-wavenumber migration and to a Kirchhoff summation. There are, however, weighting factors involved which reflect different physical situations. For instance, if the reflectivity is requested as the inversion quantity instead of the velocity perturbation, then the integral operations must be applied on data multiplied by frequency or differentiated in the time domain. Using the pressure instead of the vertical component of the particle velocity requires a factor in the migration formula that weights the data close to the apex of a diffraction, compare Eq. (21) and (30). In the acoustic case treated here, the free surface plays a role only for marine data.

The circumstance that Born inversion in the applications discussed in this paper leads to results strongly reminiscent of classical migration should not be interpreted as a shortcoming. Those methods are often based on heuristic ideas, as, for example, the exploding reflector model, rather than on a straight inversion procedure. Therefore, Born approximation must be considered as a contribution to the theoretical justification and consolidation of classical migration. Born inversion can thus rely on well-tested numerical methods and simultaneously yield true amplitudes.

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